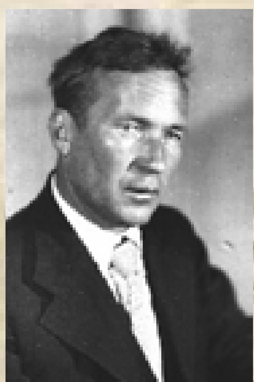
Turbulence



Vincenzo Carbone

UNIVERSITY OF CALABRIA
Department of Physics

Outline

- Turbulence: observations and main questions.
- Kolmogorov theory, energy cascade, intermittency.
- Modeling turbulence: A dynamical system approach
- I apologize, I will make an extreme confusion between laboratory and space turbulence!
- I promise to not mention words like “fractals”, “sandpiles”, etc. during this talk.

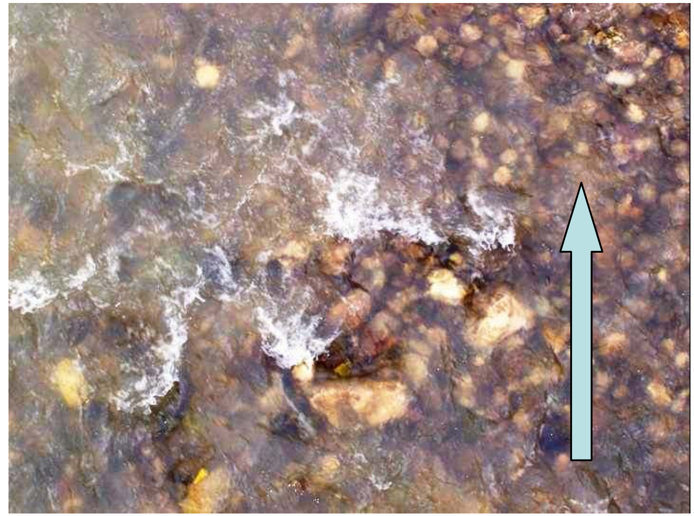
"In manibus codices, in oculis facta"

(Sentence by S. Agostino)

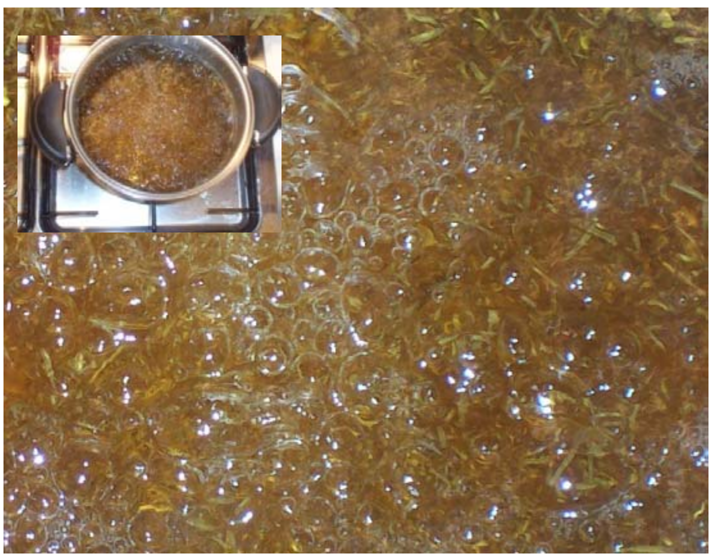
$$\partial_t u_i + u_\alpha \partial_\alpha u_i = -\partial_i P + \nu \partial^2_\alpha u_i$$

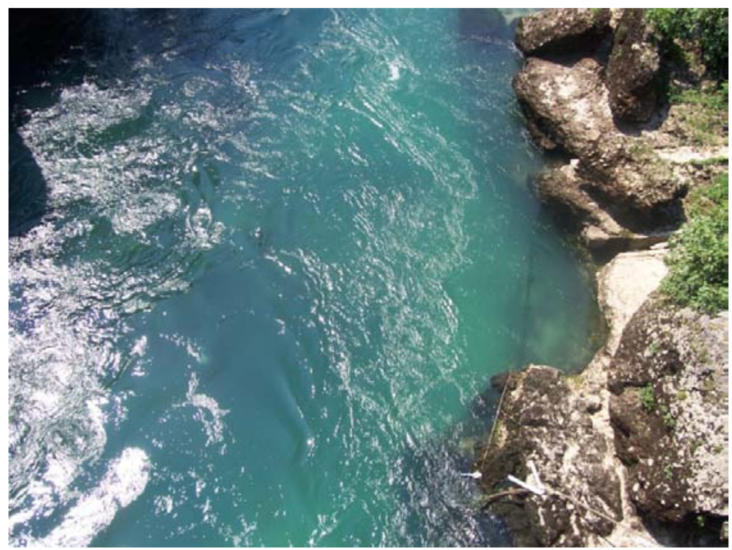
Although we know that fluid flows, and then turbulence, are described by Navier-Stokes equation, we have to look first at **real nature** to watch their **richness** and **beauty**, and realize how **dramatically difficult** is their description.

Turbulence everywhere,
A phenomenon ubiquitous in nature



flow





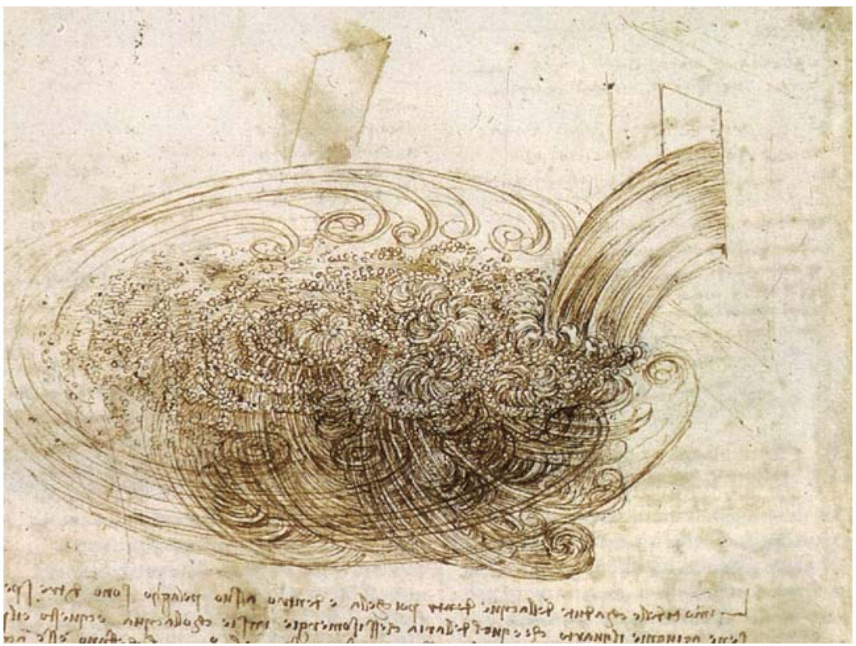
In general we don’t care about turbulence …

“Turbulence: It’s a killer ride”
(R. Liotta and L. Molly, 1977)

… apart when a nice lady asks us to keep fastened seat belt because some turbulence is approaching …

Turbulence
(*La turbolenza* since Leonardo da Vinci).
Yet in lack of a formal definition

From latin “turba” (τνρβη):
confusion (of people)

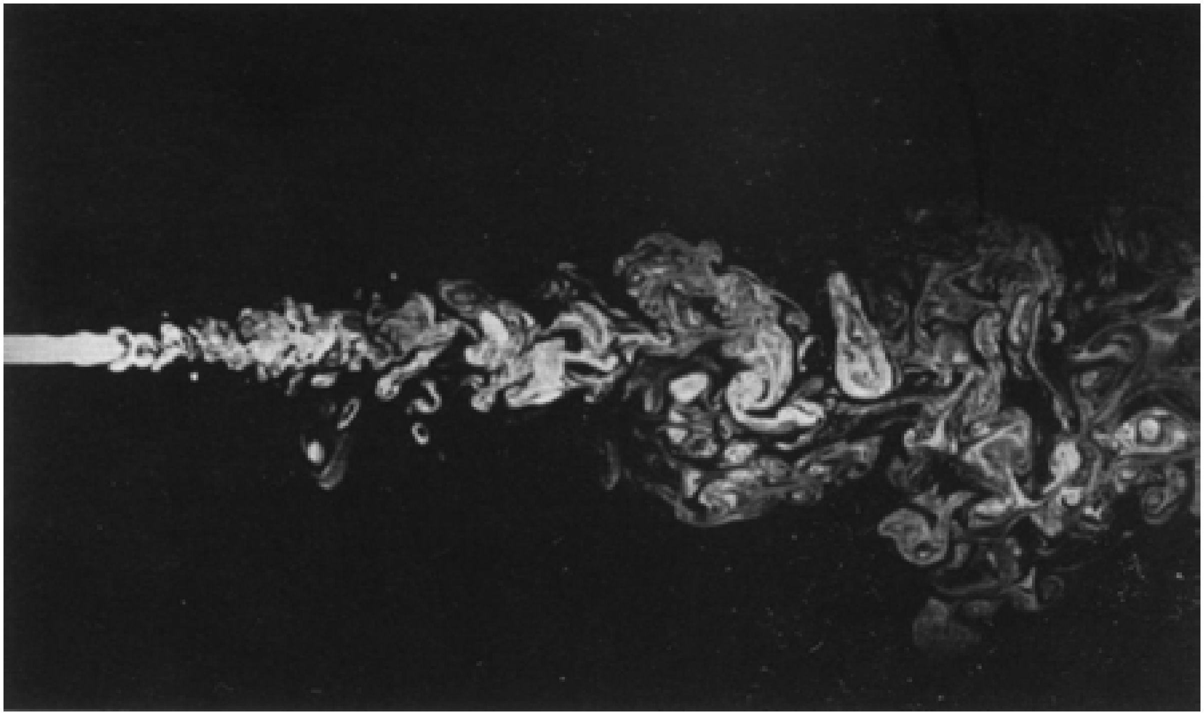


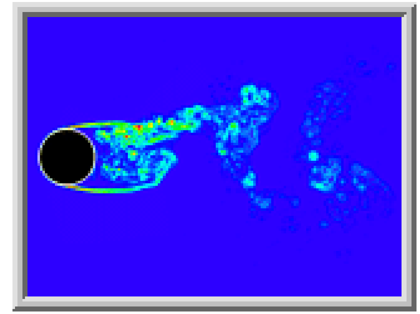
An “italian” definition:

A “turbulent boy” in all Italian schools is a young fellow who rebels against ordered schemes. Following the same line, “turbulent” is called the behavior of a flow which (apparently) rebels against deterministic rules imposed by classical mechanics.

R. Bruno & V. Carbone,
Living Rev. in Solar Phys.
(2005)

A peculiar stochastic process: strange mixing of order and chaos



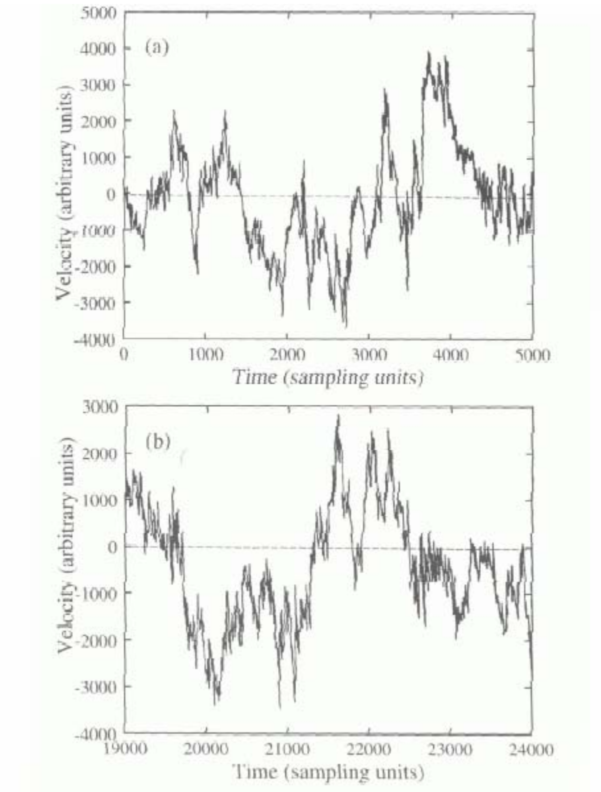


Main features:
1) Randomness both in space and time
2) Turbulent "structures" (eddies) on all scales
3) Unpredictability and instability to very small perturbations

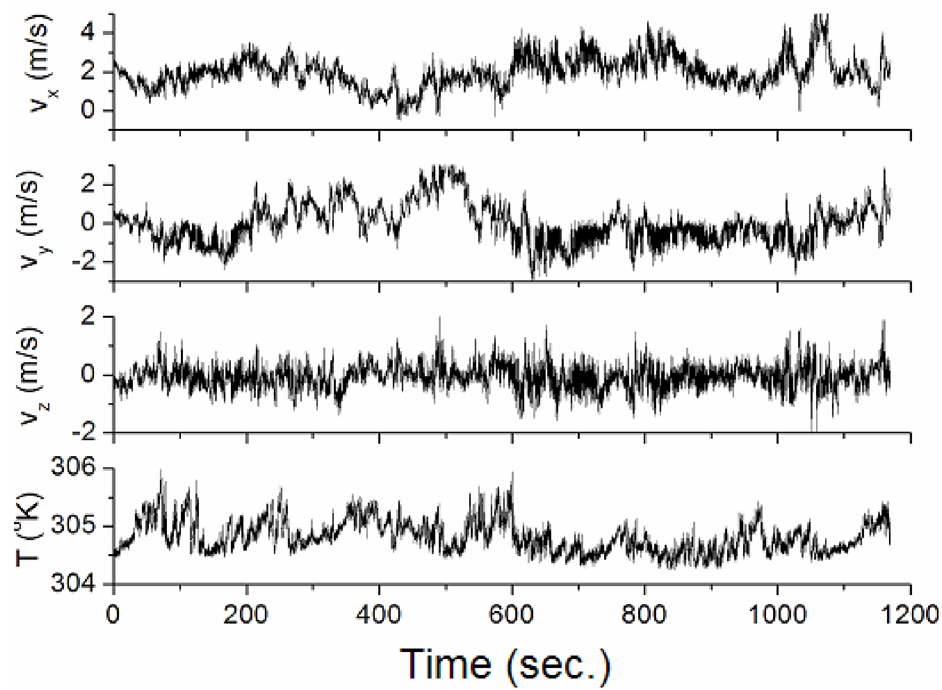
Turbulence is far from a sequence of random numbers with a well defined spectrum and uncorrelated phases. You cannot reproduce a "turbulent field" by putting at random sand on a table!

Fluid turbulent samples

The velocity trace recorded in two different points within the flow has the same global “stochastic” behavior (gaussian statistics), but local dynamic looks to be completely different

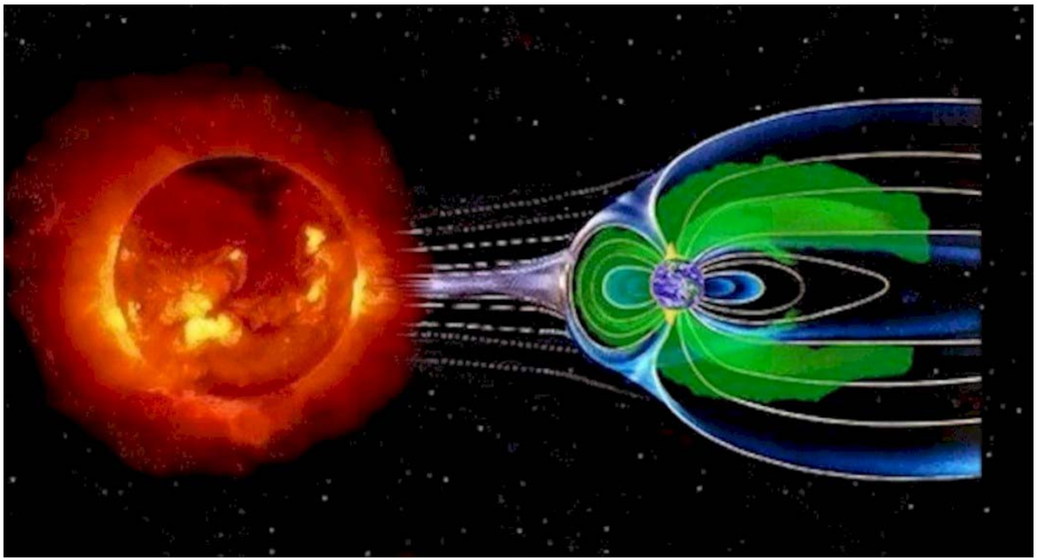


Laboratory wind tunnel



Atmospheric turbulence

The solar atmosphere extends in the interplanetary space, thus generating a turbulent flow:
THE SOLAR WIND

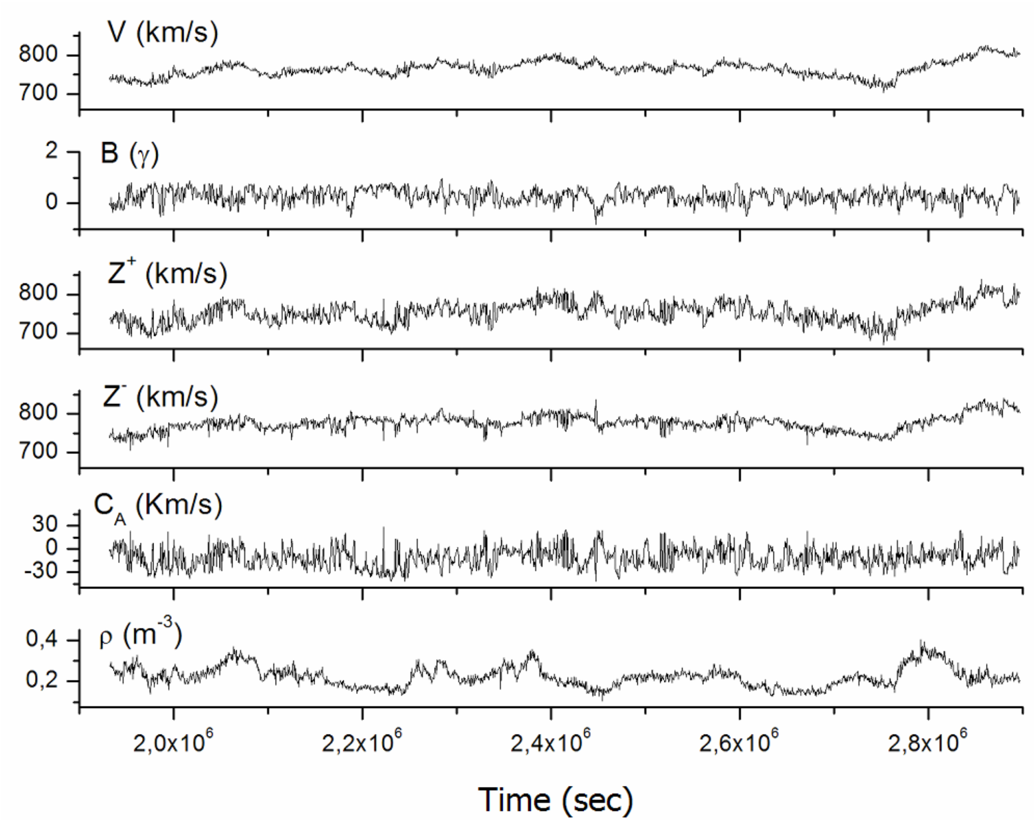


In 1957 the first spacecrafts flying in space measured turbulent fluctuations. The solar wind is a rather peculiar turbulent flow because it is essentially made by charged particles, say mainly protons and electrons (PLAMA).

Turbulence in space is rather complicated, because the fluid motion generates currents and magnetic fields that interact with the fluid flow. This is the Magneto-Hydro-Dynamic (MHD) turbulence.

50 years later the first space flights, in 2007 we celebrated the International Heliophysics Year.

The solar wind as a wind tunnel



$$z_i^{\pm} = v_i \pm b_i = v_i \pm \frac{B_i}{\sqrt{4\pi\rho}}$$

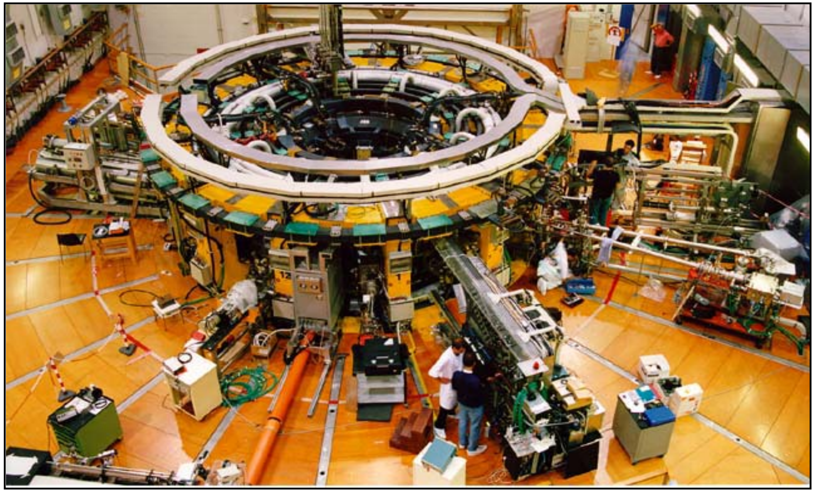
In situ measurements of high amplitude fluctuations for all fields (velocity, magnetic, temperature…)
A unique possibility to measure low-frequency turbulence in plasmas over a wide range of scales.

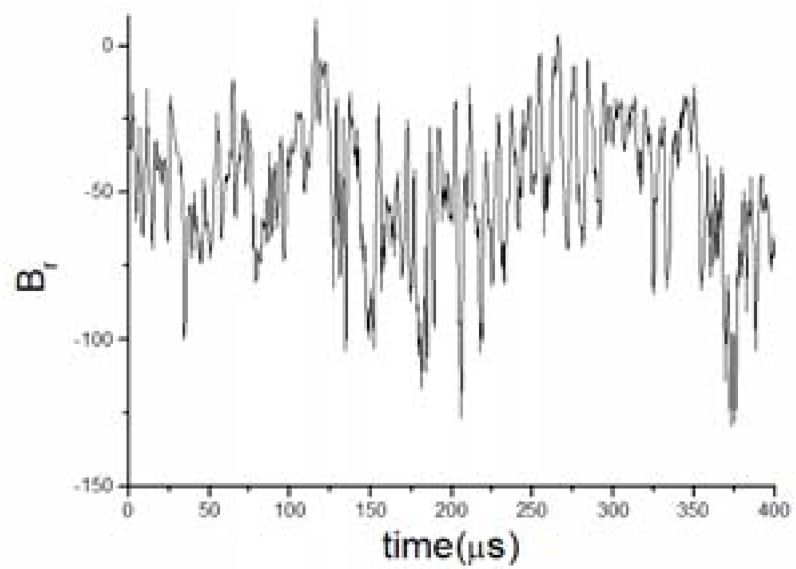
For a review:
R. Bruno & V. Carbone, Living Review in Solar Physics, (2005)
http://www.livingreview.org

An updated version will be available (hopefully) on March 2008

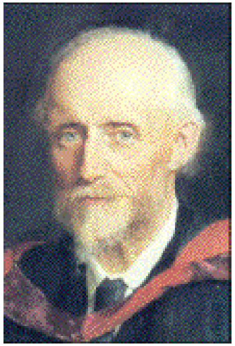
Turbulence in plasmas: laboratory experiments

Plasma generated for nuclear fusion, confined in a reversed field pinch configuration. High amplitude fluctuations of magnetic field, measurements (time series) at the edge of plasma column, where the toroidal field changes sign.





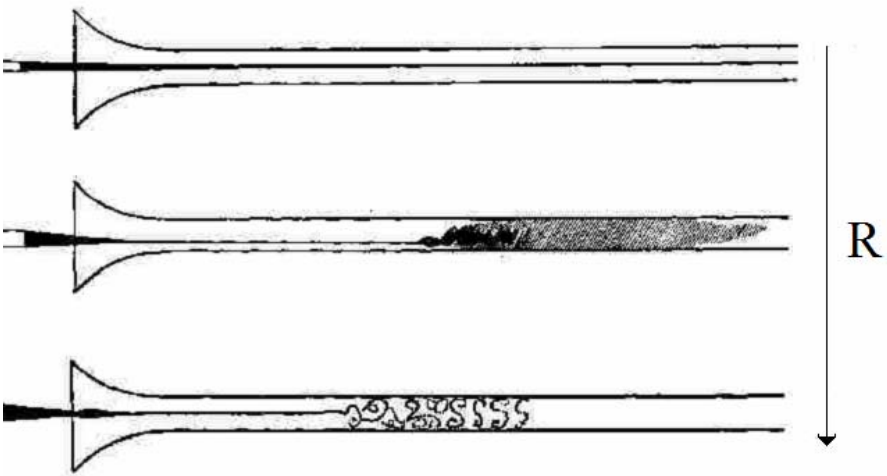
Magnetic and electrostatic turbulence data from RFX (Padua) Italy

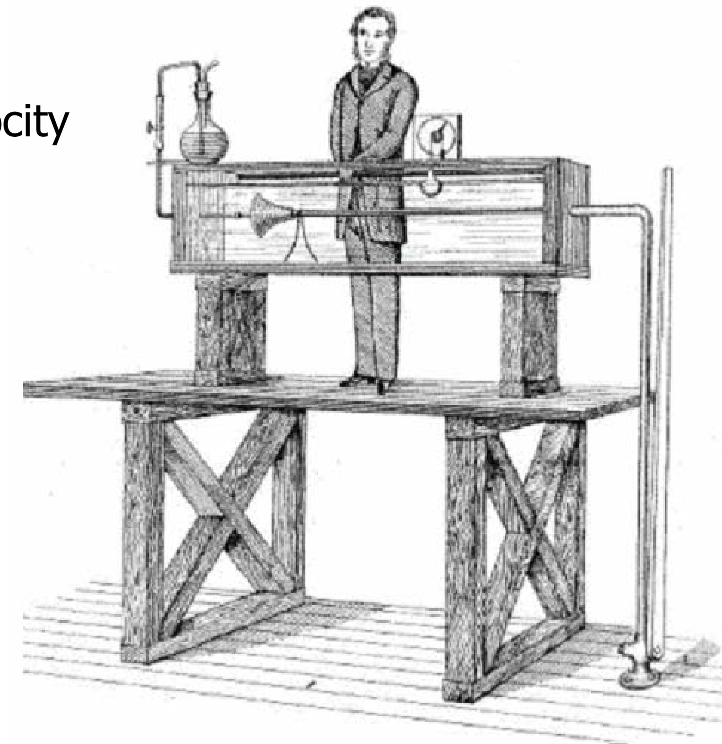


Osborne Reynolds: quantitative experiments

$$\boxed{R = \frac{UL}{\nu}}$$

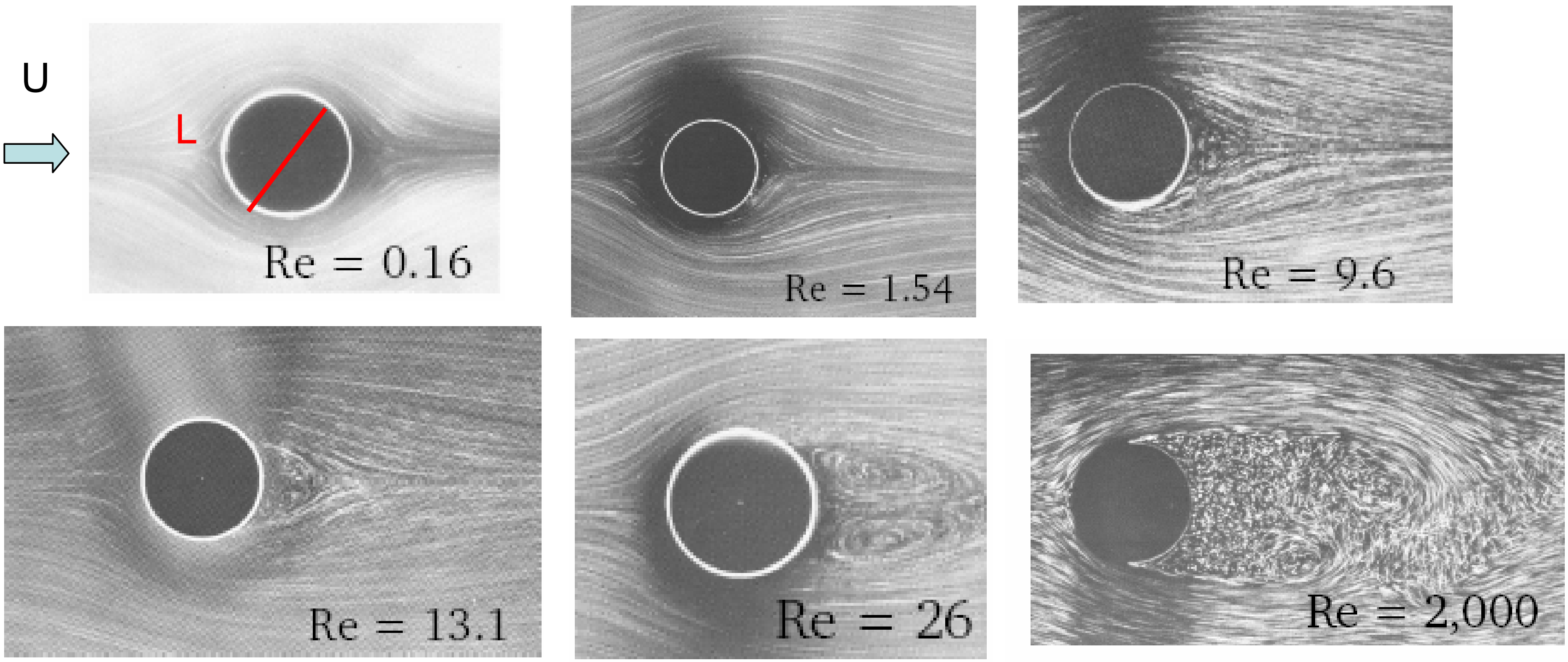
υ kinematic viscosity
U large-scale characteristic fluid velocity
L characteristic length





Osborne Reynolds noted that the dynamics is determined ONLY by a combination of characteristic parameters. As R increases the system becomes turbulent. → **Reynolds number is the control parameter**

Flow past an obstacle becomes turbulent



$R = 10^4$

Laboratory

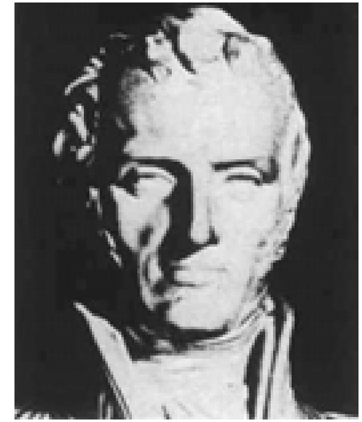
$R = 10^6$

Atmosphere

$R \approx 10^{10}$

Astrophysical fluid flows

Turbulence is a characteristic of the FLOW and is described by Navier-Stokes equation



$$\partial_t u_i + u_\alpha \partial_\alpha u_i = -\partial_i P + \nu \partial_\alpha^2 u_i$$

Nonlinear

Dissipative

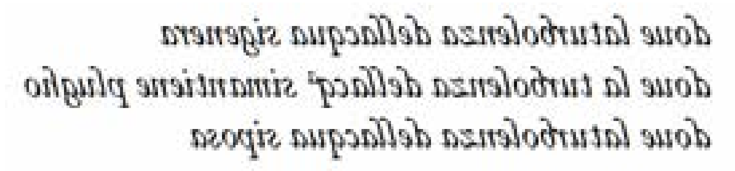
Incompressible
Navier-Stokes equation
u $\rightarrow$ velocity field
P $\rightarrow$ pressure
ν $\rightarrow$ kinematic viscosity

$$\boxed{R = \frac{\text{Nonlinear}}{\text{Dissipative}} \approx \frac{UL}{\upsilon}}$$

Turbulence is the result of nonlinear dynamics (fluid flow) of Navier-Stokes equations.

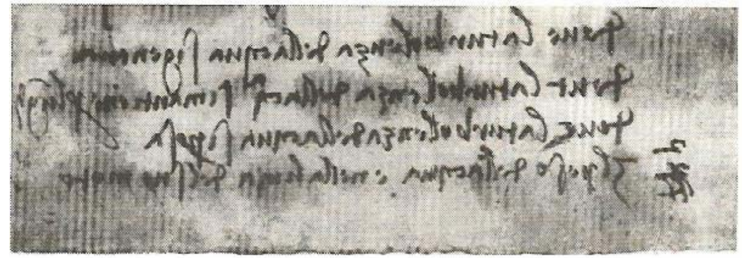
Three main questions from Leonardo

(Codice Atlantico)

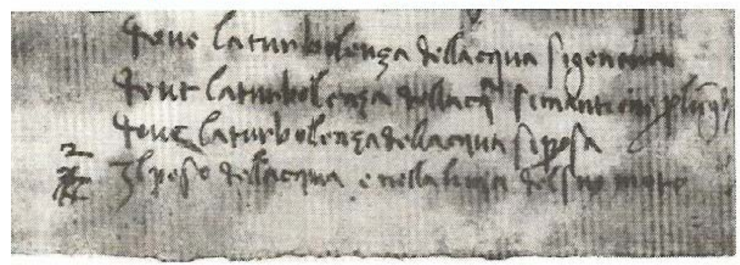


After mirror reflection

doue laturbolenza dellacqua sigenera
doue la turbolenza dellacq^{a} *simantiene plugho*
doue laturbolenza dellacqua siposa



dopo la riflessione speculare



Three main questions

1) Where the turbulence of water is generated
2) Where the turbulence of water maintains for long
3) Where the turbulence of water comes to rest

?

The exact solution worth $1M

CLAY MATHEMATICS INSTITUTE

Dedicated to increasing and disseminating mathematical knowledge

MILLENNIUM PRIZE PROBLEMS

Statement from the Directors and Scientific Advisory Board

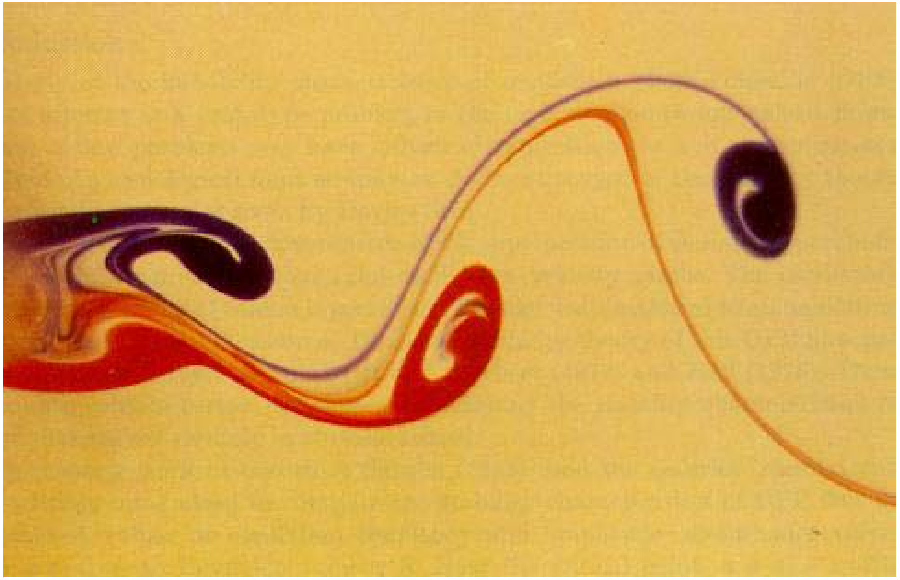
| **Birch and Swinnerton-Dyer Conjecture** | **Hodge Conjecture** | **Navier-Stokes Equations** | **P vs NP** | **Poincare Conjecture** | **Riemann** **Yang-Mills Theory** | | **Rules etc** |

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven "Millennium Prize Problems." The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI have designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each. During the Millennium meeting held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled "The Importance of Mathematics," aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

In the following, some (rough!) answers for free!

1th question

Where the turbulence of water is generated (?)



For example from Kelvin-Helmotz instability

Instability at the interface of two streams of fluid that move with different velocities

Real experiment

Numerical simulation

Turbulence and deterministic rules

Equations that describes turbulence are time invariant, they describe **deterministic phenomena**.

Laplace:

The knowledge, at a given instant of time, of ALL forces in nature and the situation of ALL particles, yields a complete predictability of dynamics.

Poincaré:

Even if we can describe ALL forces in nature, the situation of all particles at a time are know only APPROXIMATELY. Very small differences in the initial conditions should give rise to big errors at future times. Predictability is (practically) impossible.

Turbulence seems to violate the deterministic law of Laplace, even if equations are deterministic. It seems to be described in the framework of Poincarè phenomena. Very small perturbations yields unpredictability.

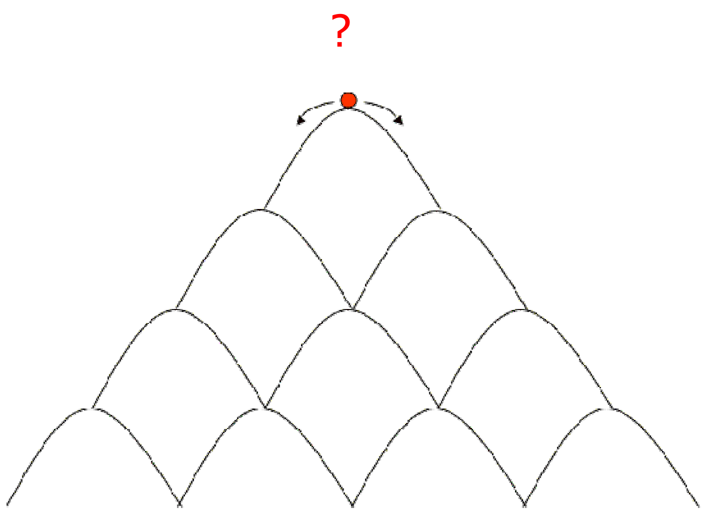
Turbulence: the main phenomenon of deterministic chaos

Some nonlinear phenomena are described by deterministic equations that are extremely sensitive to initial conditions.

Turbulence born because equations are nonlinear and extremely sensitive to initial conditions

DETERMINISTIC CHAOS (contradiction in terms!) = extreme sensitivity from initial conditions. Consequence → unpredictability

In non chaotic systems unpredictability is limited to some very peculiar initial conditions (unstable fixed point). The point needs an aid from a small fluctuations. Once the point decides the side, we can predict the future (left or right).

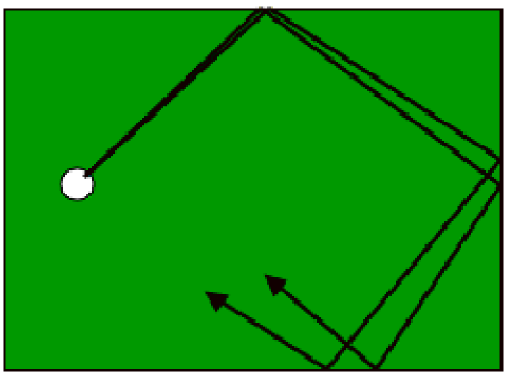


Chaos → unpredictability

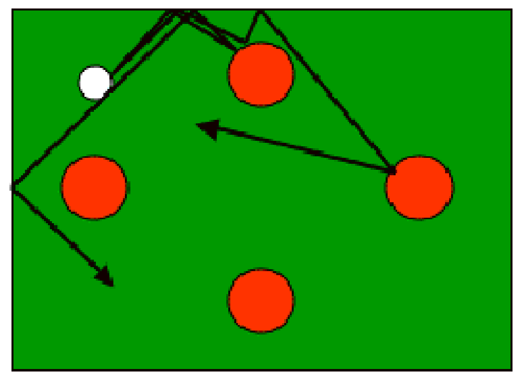
In chaotic systems unpredictability is intrinsic to the system, that is the system is "unstable" for almost all initial conditions. In chaotic systems the distance between two nearby trajectories diverges exponentially: example the Lorentz billiard.

$$D(t) = D(0) + At$$

$$D(t) = D(0)\, e^{\lambda t}$$



Predictable



Unpredictable

Two competing models for the onset of turbulence: Landau vs. Ruelle & Takens

1) Landau:
turbulence appears at the end of an infinite serie of Hopf bifurcations to quasi-periodic motion as R increases, each adding an incommensurable frequency to the flow

The more frequencies →
The more stochasticity

Chaos is absent

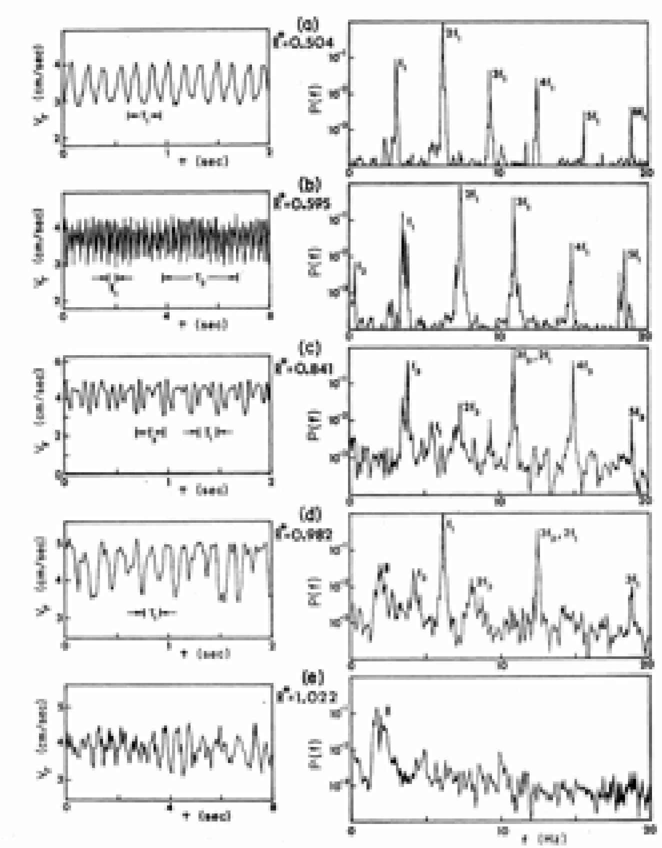
2) Ruelle & Takens:
incommensurable frequencies cannot coexist, the motion becomes rapidly aperiodic and turbulence suddenly will appear, just after three (or four) bifurcations.

The system lies on a subspace of the phase space: a "strange attractor".

Chaos is present

Accurate experiments on turbulence (not engineering but physicists!)

Gollub & Swinney, Phys. Rev. Lett., 1975



Onset of Turbulence in a Rotating Fluid*

J. P. Gollub†‡ and Harry L. Swinney
Physics Department, City College of the City University of New York, New York, New York 10031
(Received 17 July 1975)

Light-scattering measurements of the time-dependent local radial velocity in a rotating fluid reveal three distinct transitions as the Reynolds number is increased, each of which adds a new frequency to the velocity spectrum. At a higher, sharply defined Reynolds number all discrete spectral peaks suddenly disappear. Our observations disagree with the Landau picture of the onset of turbulence, but are perhaps consistent with proposals of Ruelle and Takens.

After the appearance of two incommensurable frequencies, the spectrum becomes broad abruptly

This very basic observation disagree with the Landau picture and seems to be in agreement with the idea by Ruelle & Takens.

But an example of "strange attractor" ?

E.N. Lorenz, 1963

Edward Lorenz in 1963 derived a Galerkin approximation with only three modes to get a simplified model of convective rolls in the atmosphere (2D).

$$\partial_t \nabla^2 \psi + [\partial_z \psi \partial_x - \partial_x \psi \partial_z] \nabla^2 \psi = \nu \nabla^2 \nabla^2 \psi - g\alpha \partial_x T$$

$$\partial_t T + [\partial_z \psi \partial_x - \partial_x \psi \partial_z] T = -\beta \partial_x \psi + \chi \nabla^2 T$$

$$\begin{aligned} \psi(x,z,t) &= A_{1,1}(t) \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi z}{d}\right) \\ T(x,y,z) &= B_{1,1}(t) \cos\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi z}{d}\right) + B_{0,2}(t) \sin\left(\frac{2\pi z}{d}\right) \end{aligned}$$

Retains only three modes in a Fourier expansion

$$\begin{aligned} \frac{dA_{1,1}}{dt} &= -\frac{\nu\pi^2}{d^2}\left(\frac{d^2}{L^2}+1\right) A_{1,1} + \frac{g\alpha d^2}{\pi L(1+d^2/L^2)} B_{1,1} \\ \frac{dB_{1,1}}{dt} &= \frac{\pi^2}{Ld} A_{1,1} B_{0,2} + \frac{\beta\pi}{L} A_{1,1} - \frac{\chi\pi^2}{d^2}\left(\frac{d^2}{L^2}+1\right) B_{1,1} \\ \frac{dB_{0,2}}{dt} &= -\frac{\pi^2}{Ld} A_{1,1} B_{1,1} - \frac{4\pi\chi}{d^2} B_{0,2} \end{aligned}$$

r is the ratio between the Raylaigh number and the critical one for convection (r > 1 implies convection);

$$\begin{aligned} A_{1,1}(t) &= \frac{\sqrt{2}\chi}{a}(1+a^2) x(t) \\ B_{1,1}(t) &= \frac{\sqrt{2}}{\pi}\frac{Ra^*}{Ra} \beta d y(t) \\ B_{0,2}(t) &= -\frac{\beta d}{\pi}\frac{Ra^*}{Ra} z(t) \end{aligned}$$

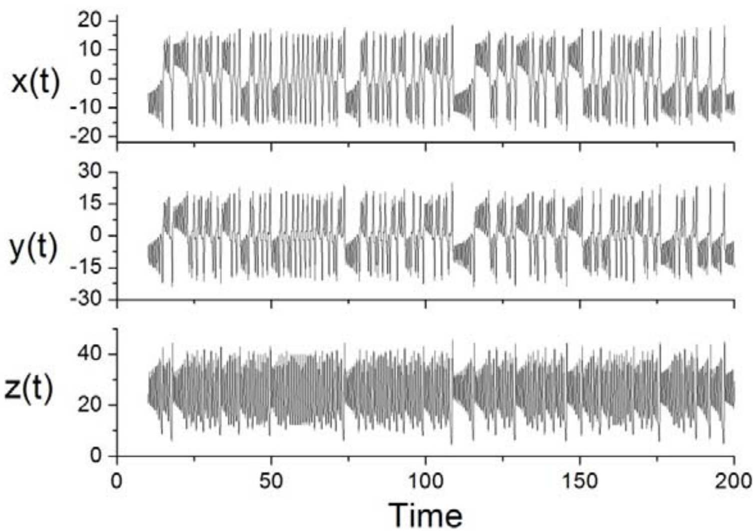
$$\begin{aligned} \frac{dx}{dt} &= -\sigma x + \sigma y \\ \frac{dy}{dt} &= -xz + rx - y \\ \frac{dz}{dt} &= xy - bz \end{aligned}$$

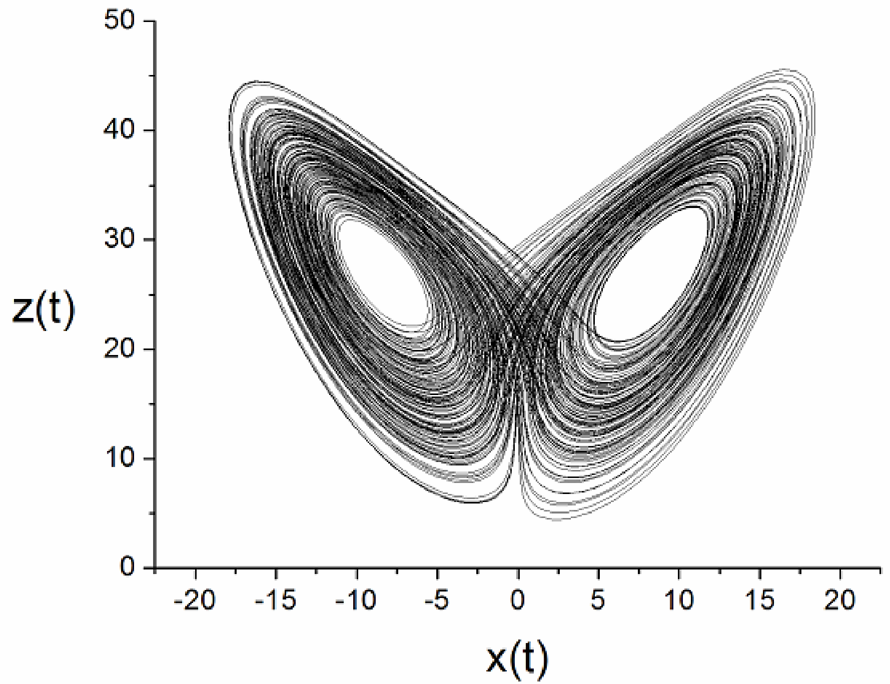
b is a geometric factor (aspect ratio);

σ is the Prandtl number (ratio between kinematic viscosity and thermal diffusivity)

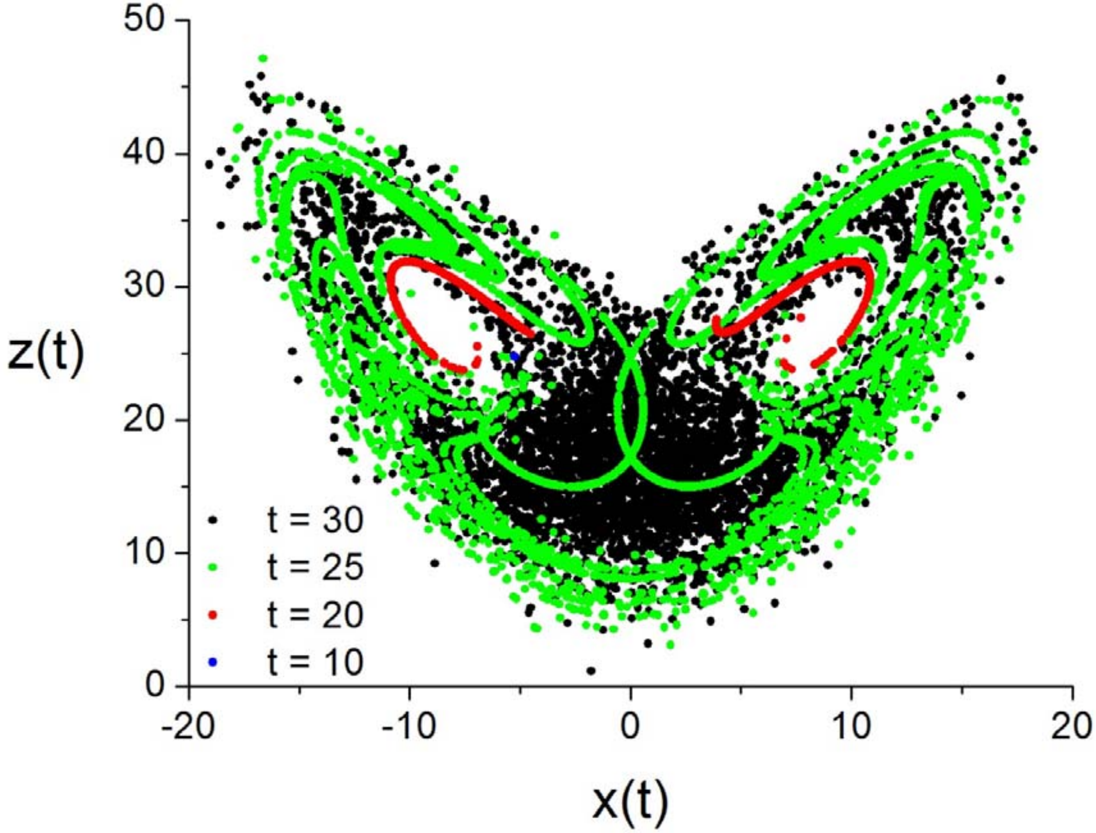
An example of "strange attractor"

The trajectories of the system, for certain settings, never settle down to a fixed point, never approach a stable limit cycle, yet never diverge to infinity. The phase space is contracting to a set of dimension zero (dissipative system!),… trajectories are "condemned" to wonder forever within the contracting finite portion of the phase space without intersections (deterministic system).





Strangeness:
Extreme sensitivity to initial conditions



5000 trajectories from different initial conditions. The initial conditions are confined in a square of amplitude 10^{-5}

Chaotic dynamics from Navier-Stokes equations: nonlinear evolution →1D map (discrete times)

$$\frac{\partial \mathbf{u}}{\partial t} = -\left[(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla P\right] + \nu\nabla^2\mathbf{u} + \mathbf{f}$$

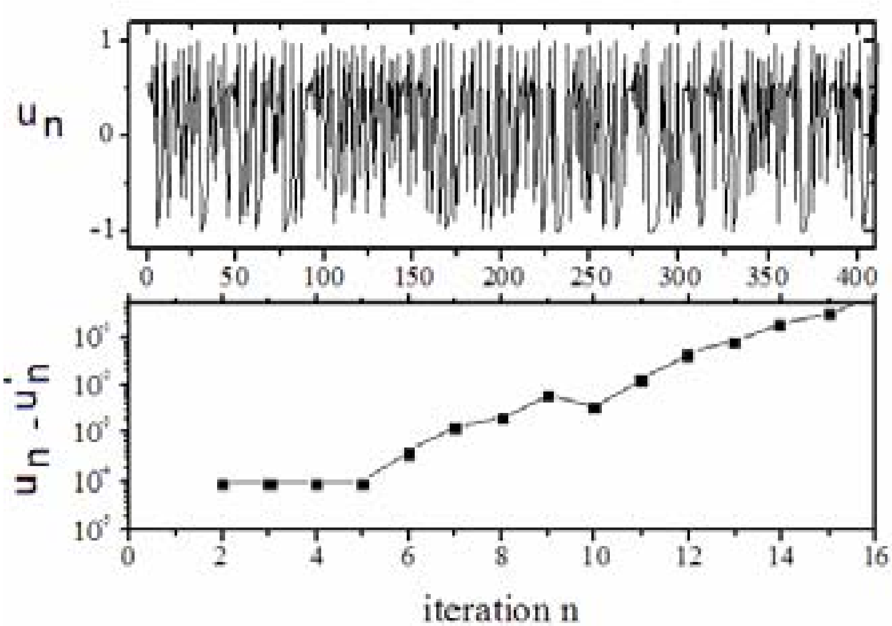
$$\boxed{u_{n+1} - u_n = -2u_n^2 - u_n + 1}$$

"poor man's NS equation"

Let us add an external forcing term to restore turbulence

$$u_{n+1} = 1 - 2u_n^2$$

$$u_n \in [0,1]$$



$$u_{n+1} = T(u_n) \quad \text{nonlinear map}$$

1) Stochastic behaviour (randomness)
2) No predictability: two nearby trajectories diverge exponentially (sensitivity to initial conditions)

Chaotic dynamics in a (very simple) deterministic system

Sensitivity to initial conditions ?

$$u_{n+1} = 1 - 2u_n^2$$

A transformation
leads to the tent map
(stretch & fold)

$$u_n = sin(\pi x_n - \pi / 2)$$

$$\Rightarrow x_{n+1} = \begin{cases} 2x_n & x_n \in [0,1/2] \\ 2(1-x_n) & x_n \in [1/2,1] \end{cases}$$

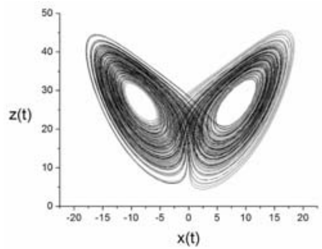
Numbers written in binary format

$$x_n = 0.a_n(1)a_n(2)\ldots a_n(i)\ldots$$
$$a_n(i) = [0,1]$$

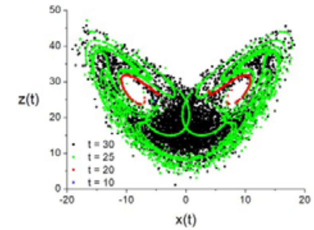
Iterates of the tent map
lead to the (from right to left)
"Bernoulli shift"

$$a_{n+1}(i) = \begin{cases} a_n(i+1) & a_n(1) = 0 \\ 1 - a_n(i+1) & a_n(1) = 1 \end{cases}$$

A small uncertainty surely will grows in time !
No predictability on arbitrarily long times
Sensitivity of system to every small perturbations



Why chaos is “interesting”?
Chaotic dynamic leads to stochasticity



$$x_{n+1} = T \otimes T \otimes \ldots T(x_0) = T^n(x_0)$$

Apply the map n times

As a consequence of the chaoticity, the trajectory of a **SINGLE** orbit covers **ALL** the allowed phase space

Ergodic theorem: Let f(x) an integrable function, and let $f(T^n(x_0))$ calculated over all iterates of the map. Then for almost all x_0

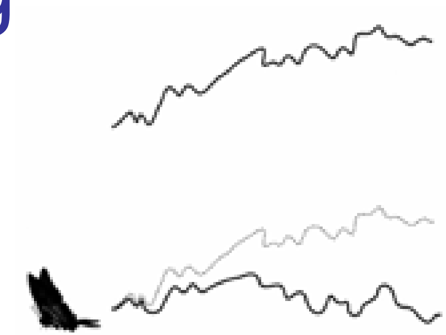
$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} f(T^n(x_0)) = \int_0^1 f(x)dx$$

$$\langle f \rangle_{TIME} = \langle f \rangle_{ENSEMBLE}$$

Here the ensemble is generated by the chaotic dynamics, from the uniform measure in [0,1].

Turbulence and unpredictability in the atmosphere: the "butterfly" effect and weather forecasting

Weather forecasting for long times?



Per colpa di un chiodo si perse lo zoccolo
per colpa di uno zoccolo si perse il cavallo
per colpa del cavallo si perse il cavaliere
per colpa di un cavaliere si perse il messaggio
per colpa di un messaggio si perse la battaglia
per colpa di una battaglia si perse il regno

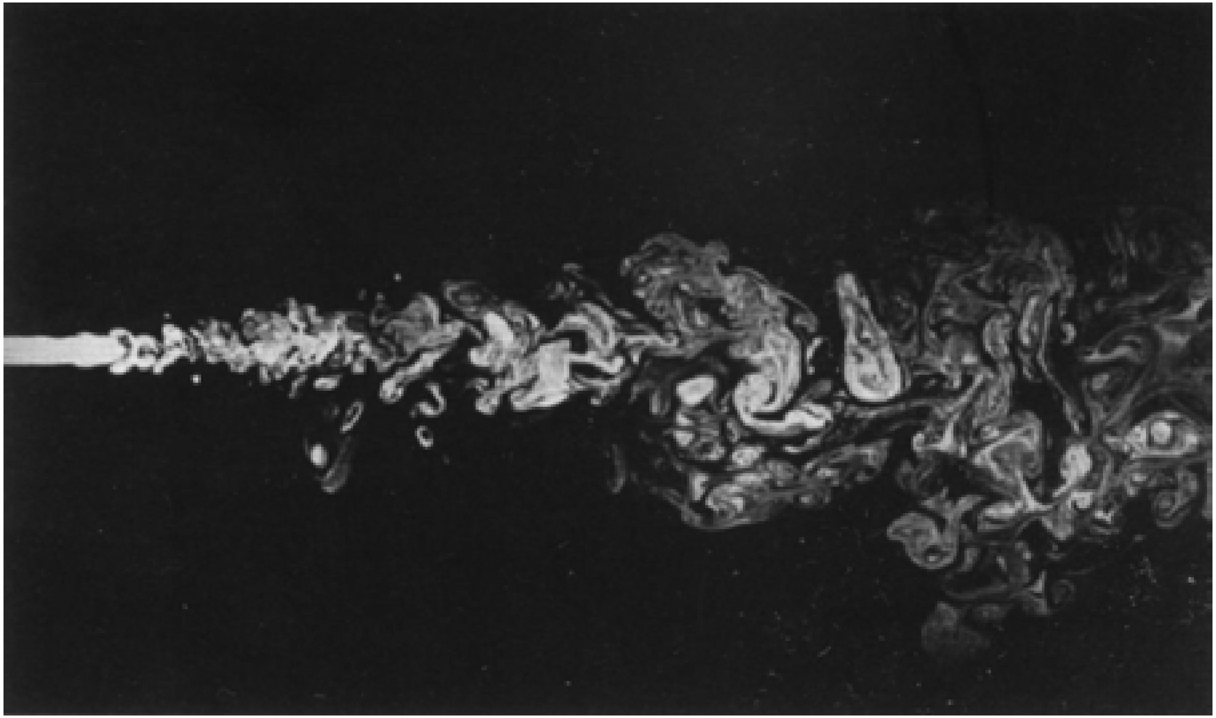
G. Herbert

" Lontana previsione a lungo dura, vicina prevision meno sicura"

Detto popolare

2th question

Where the turbulence of water maintains for long (?)



Where the problem comes from

$$\partial_t u_i + u_\alpha \partial_\alpha u_i = -\partial_i P + \nu \partial_\alpha^2 u_i$$

$$\tau_\ell \sim \frac{\ell}{u_\ell} \qquad \tau_D \sim \frac{\ell^2}{\nu}$$

From Navier-Stokes equations we can find two characteristic times for the two basic processes: a convective (eddy-turnover) time and a diffusive (dissipative) time

$$R = \frac{\tau_D}{\tau_L} = \frac{UL}{\nu}$$

Their ratio, at the largest scale L, is the Reynolds number

$$\epsilon_D \sim \frac{U^2}{\tau_D} \sim \frac{U^2 \nu}{L^2}$$

$$\epsilon_L \sim \frac{U^2}{\tau_L} \sim \frac{U^3}{L} = R\epsilon_D$$

At the largest scale L the energy injection rate (per unit mass) turns out to be R times greater than the energy dissipation rate

The turbulent system cannot dissipate the whole energy injected at the largest scale L, just a fraction of it. How the system can dissipate the excess energy ?

Example: the swimmer

$$U \approx 1 \ m/\sec$$

$$L \approx 1\,\mathrm{m}$$

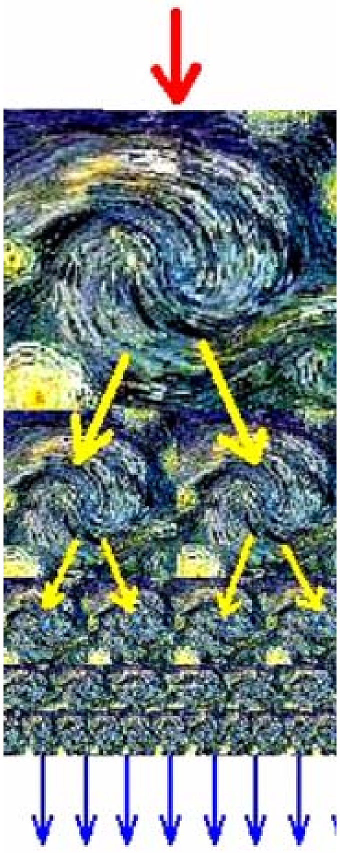
$$\boxed{\begin{array}{l} \varepsilon_D \approx 10^{-6} \\ \varepsilon_L \approx 1 \end{array}}$$

R = 10^6

The swimmer puts 1 Watt/Kg in the system, and reaches a Reynolds number of the order of 10^6. The swimmer must produce turbulence

Turbulence: how fluid flows at high Reynolds number can dissipate energy !!!!

Injection of energy



Dissipation of energy

The dissipation is effective only at very small scales: the system dissipates energy simply through a transfer of energy to small scales (eddies distortion) → **energy cascade**

The Richardson's phenomenology: breaks down of eddies at large scales and transfer of energy to small scales.

Fourier analysis of equations

$$\frac{\partial u_\alpha(\mathbf{k},t)}{\partial t} = M_{\alpha\beta\gamma}(\mathbf{k})\sum_p u_\beta(\mathbf{p},t)u_\gamma(\mathbf{k}-\mathbf{p},t) - \upsilon k^2 u_\alpha(\mathbf{k},t) + f_\alpha(\mathbf{k},t)$$

$$M_{\alpha\beta\gamma}(\mathbf{k}) = -ik_\beta\left(\delta_{\alpha\gamma} - \frac{k_\alpha k_\gamma}{k^2}\right)$$

The evolution of the field for a single wave vector is related to fields of **ALL** other wave vectors (convolution term) for which **k** = **p** + **q**.

Infinite number of modes involved for inviscid flows

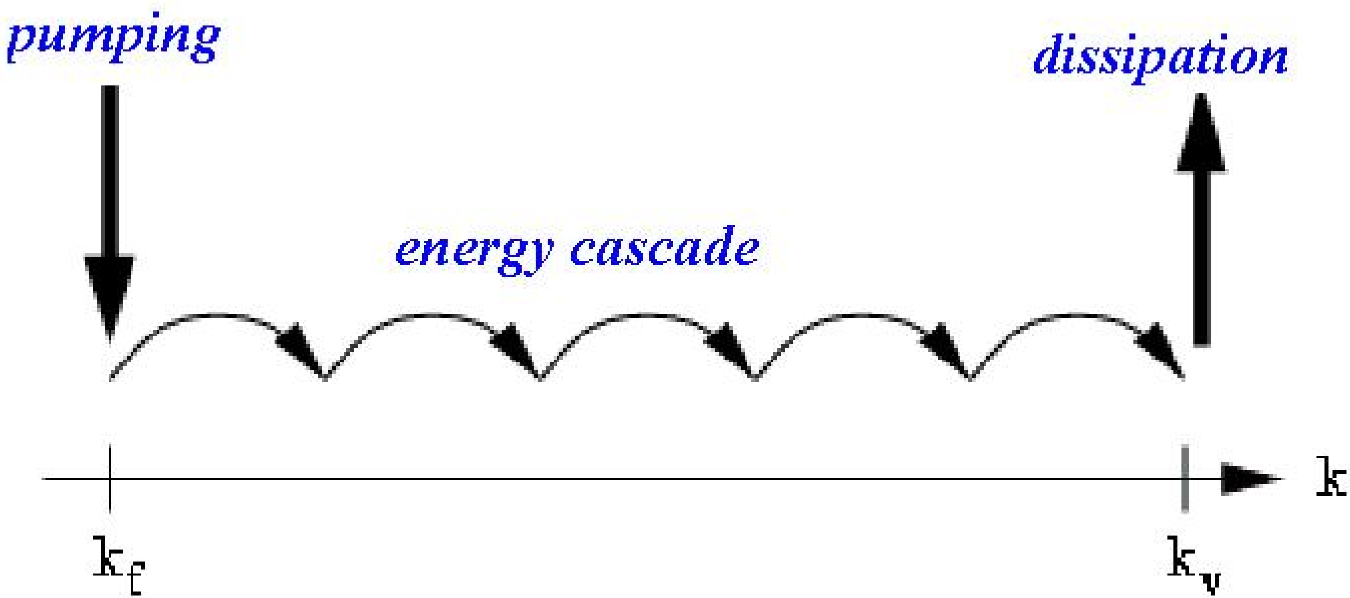
In the ideal case, the nonlinear term conserves global energy

→

The nonlinear term is responsible for a redistribution of energy over the whole set of wave vectors.

Richardson phenomenology in k-space

- Three ranges of scale (lengthscale = 1/k): energy containing, inertial (cascade), dissipative



An exact law from Navier-Stokes equations

$$\Delta u_i = \left[u_i(x+\ell) - u_i(x)\right]$$

Two-points differences along the LONGITUDINAL direction (stationary stochastc variables)

Under the conditions of homogeneity and isotropy, in the stationary state a Yaglom's relation can be derived from Navier-Stokes equation:

$$\left\langle \Delta u_\ell \Delta u_i^2 \right\rangle = 2\nu \frac{\partial}{\partial \ell}\left\langle \Delta u_i^2 \right\rangle - \frac{4}{3}\langle \varepsilon \rangle \ell$$

< ε > averaged energy dissipation rate

In the **inertial range** (a FORMAL definition of inertial range!) $\Rightarrow$

$$\boxed{\left\langle \Delta u_\ell^3 \right\rangle = -\frac{4}{5}\langle \varepsilon \rangle \ell}$$

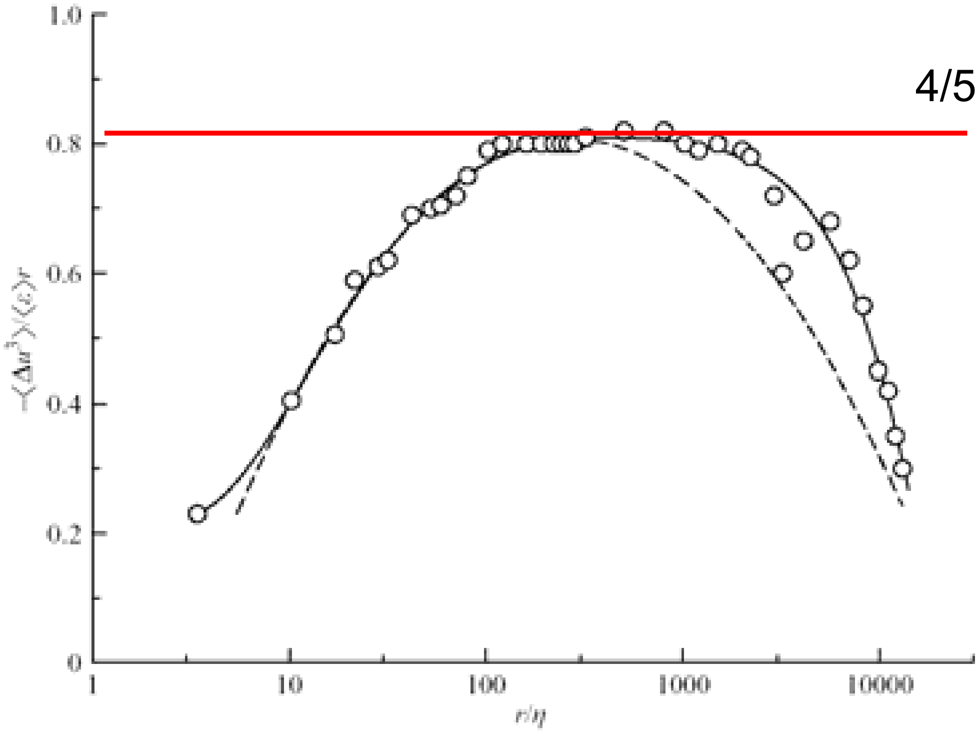
4/5-Kolmogorov law

Differences of the streamwise component of velocity between two points along the longitudinal direction. Characteristic fluctuations across eddies at the scale ℓ

a) The negative sign IS CRUCIAL!!! (irreversibility) → energy cascade
b) The third-order moment of fluctuations is related to the energy dissipation rate and is different from zero → turbulence MUST shows some nongaussian features, at least within a certain range of scales

The 4/5-law in action within fluid flows

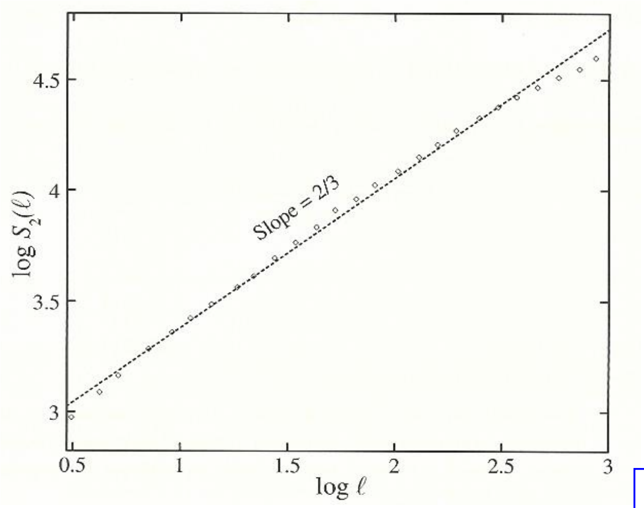
Sreenivasan & Dhruva (1998), atmospheric turbulence



Being the only exact and nontrivial relation of turbulence, the 4/5-law represents a cornerstone for modeling of turbulence. Any serious attempt to describe turbulence MUST (at least) satisfies this law.

Turbulence: the legacy of A.N. Kolmogorov (1941)

Dynamical properties of turbulence are random, but statistical properties are predictable and universal



When the flux is homogeneous along a hierarchy of vortices (similarity hypothesis) then:

$$\Delta u_\ell \approx \varepsilon^{1/3} \ell^{1/3}$$

In the inertial range

$$\boxed{\frac{1}{2}\left\langle \Delta u_\ell^2 \right\rangle = S_2(\ell) \approx \ell^{2/3}}$$

$$k \approx 1/\ell$$

$$\boxed{S_2(\ell) = \left\langle [u(x+\ell) - u(x)]^2 \right\rangle = 2\int_0^\infty E(k)\left[1 - \frac{\sin k\ell}{k\ell}\right] dk}$$

$$\boxed{E(k) \approx k^{-5/3}}$$

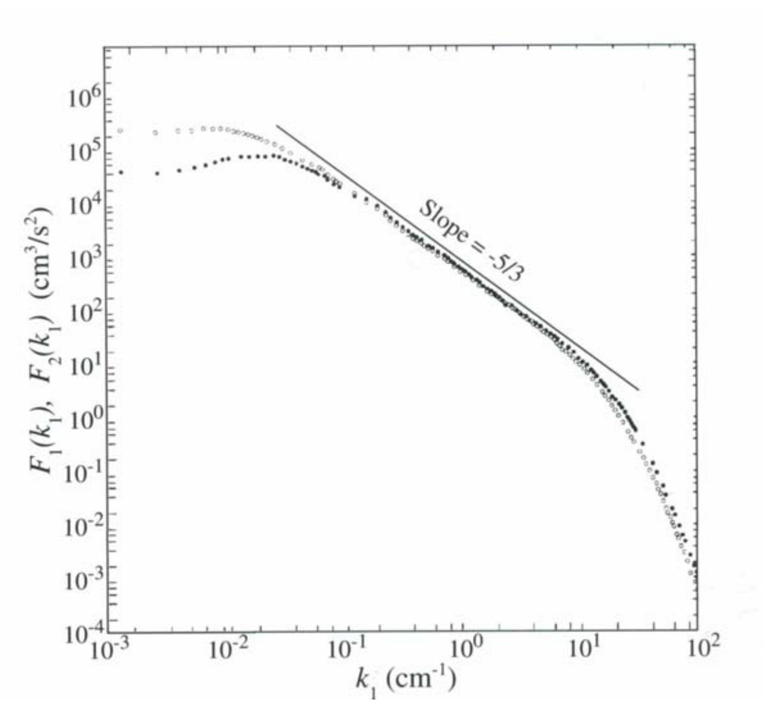
The Kolmogorov spectrum is more "famous" than the 4/5-law but (perhaps) less fundamental

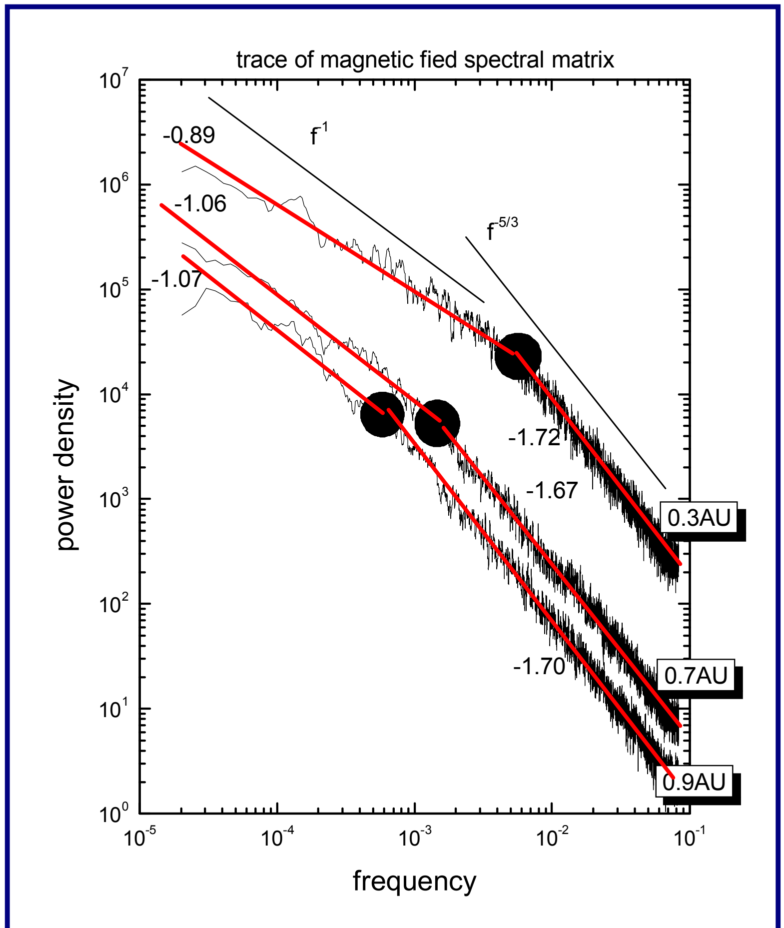
A universal energy spectrum can be observed almost in all turbulent flows !

Kolmogorov spectrum

(Low frequency) Solar wind

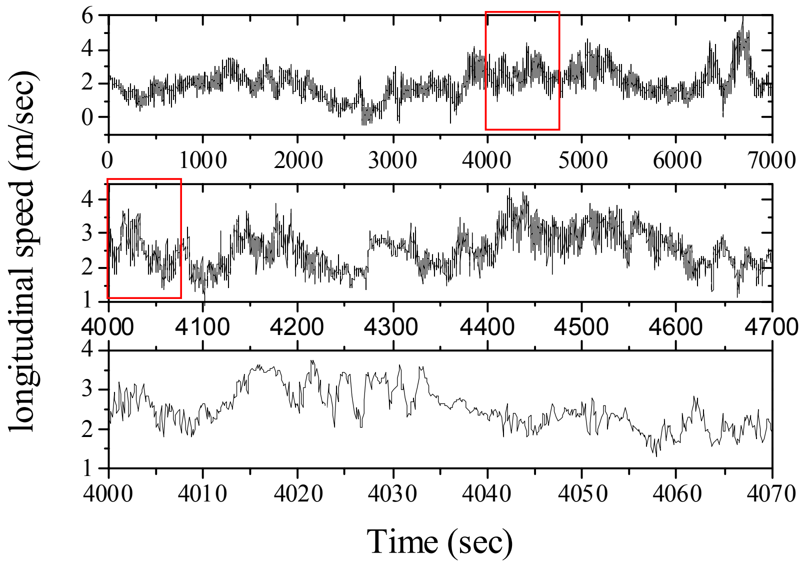
Fluid flow





Notes: dynamics vs. statistics

Atmospheric flow



1) Stochastic behaviour: the dynamics is unpredictable both in space and time.

2) Predictability is introduced at a statistical level (via the ergodic theorem and the properties of chaos!). The measured velocity field is a stochastic field with gaussian statistics.

3) On every scale details of the plots are different but statistical properties seems to be the same (apparent self-similarity).

While the details of turbulent motions are extremely sensitive to triggering disturbances, statistical properties are not (otherwise there would be little significance in the averages!)

Could turbulence be described by classical thermodynamics ? ABSOLUTELY NO!
Turbulence is close to an equilibrium system as a waterfall to a calm water in a lake.
Thermodynamic theory would predict that a normal turbulent system is many million degrees hot. Absurd.

Since turbulence must be non gaussian the 2-th order moment CANNOT plays any privileged role. We must calculate the whole set of higher-order moments

Let x a stochastic variable distributed according to a Probability Density Function (pdf) p(x), the n-th order moment is

$$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n dP(x) = \int_{-\infty}^{\infty} x^n p(x) dx \qquad \theta(k) = \int_{-\infty}^{\infty} e^{ikx} p(x) dx = \langle e^{ikx} \rangle$$

Through the inverse transform the pdf can be written in terms of moments, and moments can be obtained through the knowledge of pdf

$$p(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{-ikx} \sum_{n=0}^{\infty} \frac{(ik)^n}{n!} \langle x^n \rangle \qquad \langle x^n \rangle = i^{-n} \left. \frac{d^n \theta(k)}{dk^n} \right|_{k=0}$$

Gaussian process: the 2-th order moment suffices to fully determine pdf. High-order moments are uniquely defined from the 2-th order

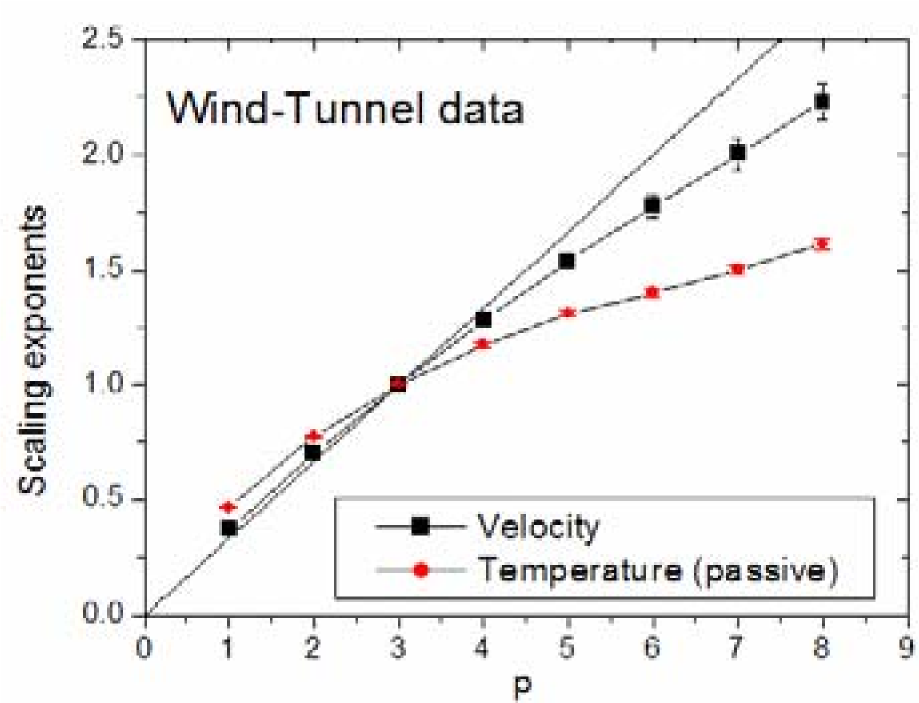
Kolmogorov conjecture: a linear scaling law for high-order moments

$$\boxed{\langle \Delta u_\ell^p \rangle = C_p \varepsilon^{p/3} \ell^{p/3}}$$

Despite the Yaglom-law and a 5/3-spectrum are observed, experiments show a strong departure from the Kolmogorov's conjecture for high-order moments

$$S_n(\tau) = \langle [u(t+\tau) - u(t)]^n \rangle \sim \tau^{\zeta_n}$$

1) u along the main flow;
2) Taylor hypothesis to transform length scales in time scales

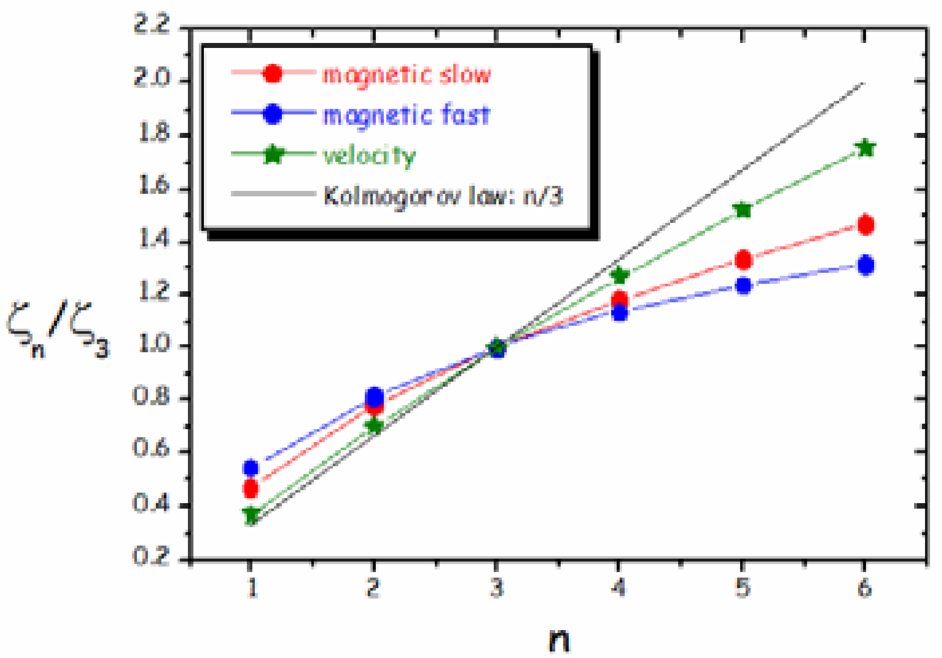


The departure has been attributed to **INTERMITTENCY** in fully developed turbulence

Fluid flows: Intermittency, measured as the distance from the Kolmogorov's linear law, is stronger for passive scalar

The same behaviour in the Solar Wind turbulence

$$S_n(\tau) = \langle [u(t+\tau) - u(t)]^n \rangle \sim \tau^{\zeta_n}$$



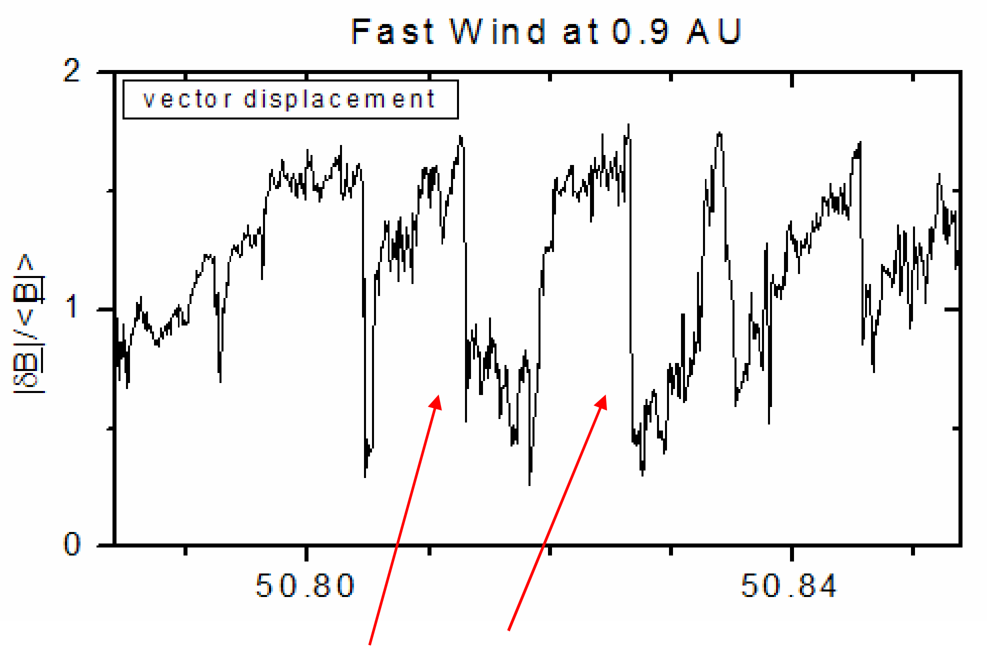
1) u along the sun-earth (longitudinal) direction;
2) Taylor hypothesis to transform length scales in time scales

Solar wind: Intermittency is stronger for magnetic field than for velocity field. Scaling laws for velocity field in the solar wind coincide with that observed in fluid flows

THIS CANNOT IMPLIES THAT THE MAGNETIC FIELD IS A “PASSIVE VECTOR”

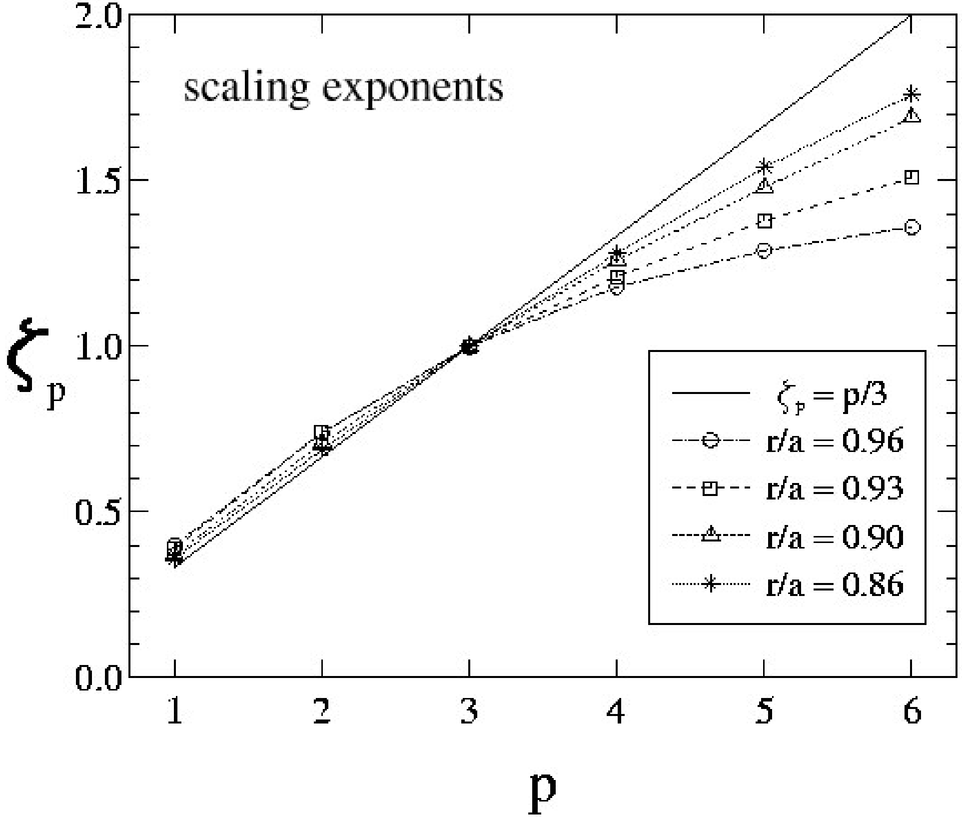
Statistics cannot prove anything, just disprove!

A very interesting example of the fact that: even if some **STATISTICAL** features of two phenomena are the same, the **DYNAMICAL** role played by quantities involved can be completely different.



Strong jumps of magnetic orientation are responsible for the strong intermittency: statistically similar to passive scalars (but dynamically different!)

Magnetic turbulence in laboratory plasma



r/a → normalized distance

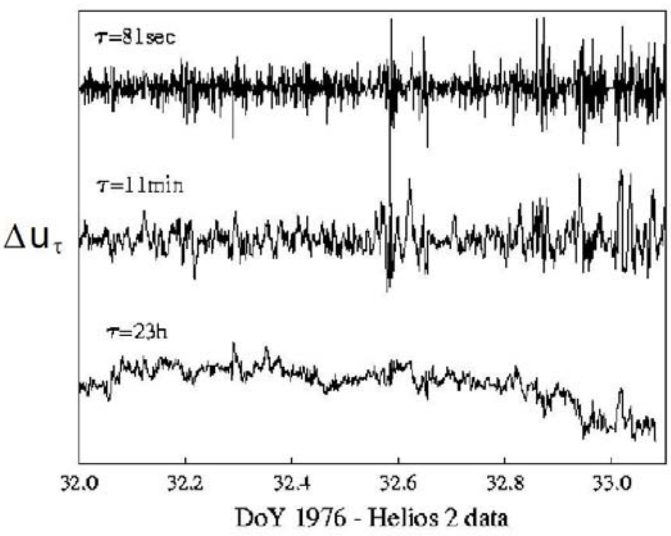
The departure from the linear scale increases going towards the wall

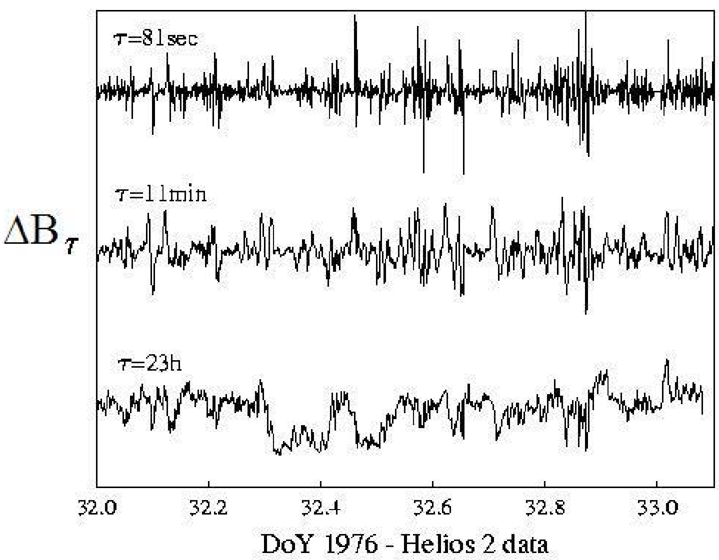
Turbulence more intermittent near the external wall

Similar to edge turbulence in laboratory fluid flows

What is “intermittent” in turbulence

Velocity and magnetic differences at three different separation times





1) A random signal at large separations;
2) Bursts of activity at smaller separations

A modified similarity hypothesis

- Landau noted that there are no physical reasons for the energy dissipation rate to be a constant.
- Kraichnan noted that, looking for scaling laws within the inertial range, we have to look at the (fluctuating) energy transfer rate at a given scale.

$$\left\langle \left(\Delta u_\ell\right)^p \right\rangle \approx \left\langle \left(\varepsilon_\ell\right)^{p/3} \right\rangle \ell^{p/3} \approx \ell^{\zeta_p}$$

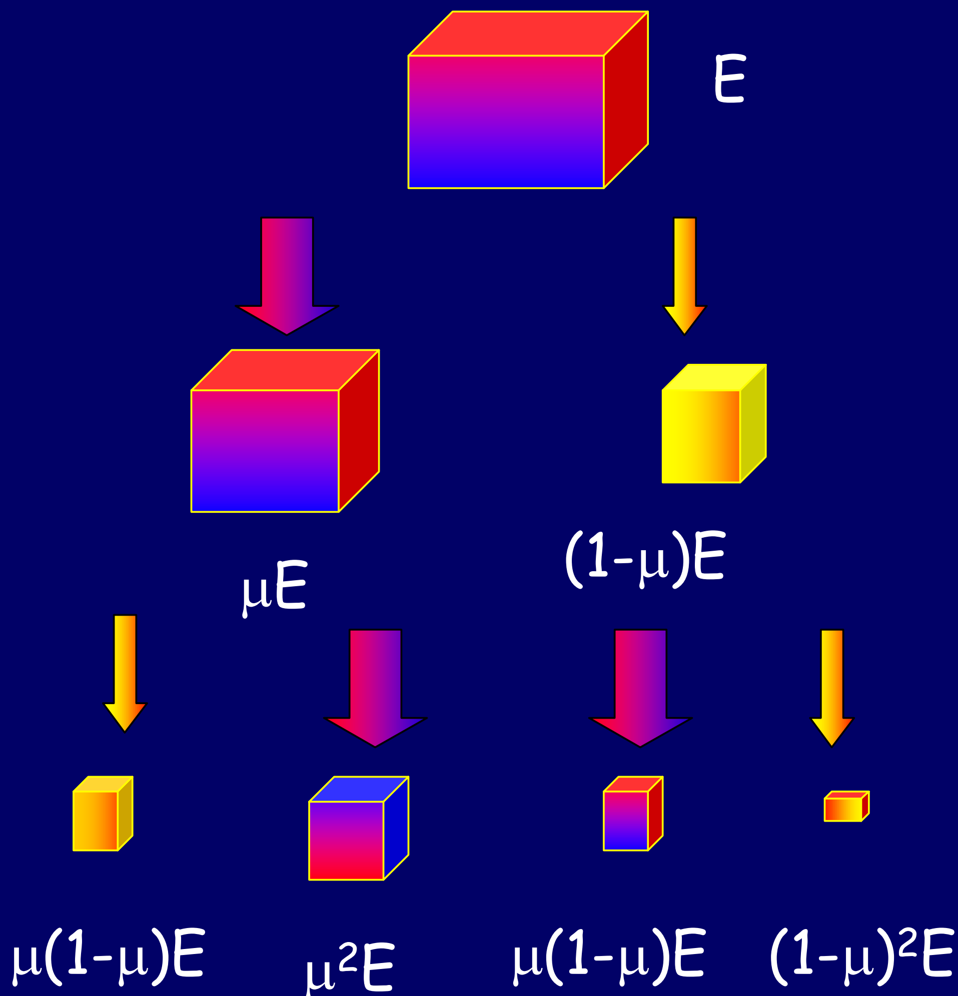
The scaling exponents of the fields depend on the scalings of fluctuations of the energy transfer rate

$$\left\langle \left(\varepsilon_\ell\right)^m \right\rangle \approx \ell^{\tau_m}$$

$$\zeta_p = p/3 + \tau_{p/3}$$

A "Pandora's box" of possible models for the energy transfer rate. Each model gives different intermittent correction

The p-model

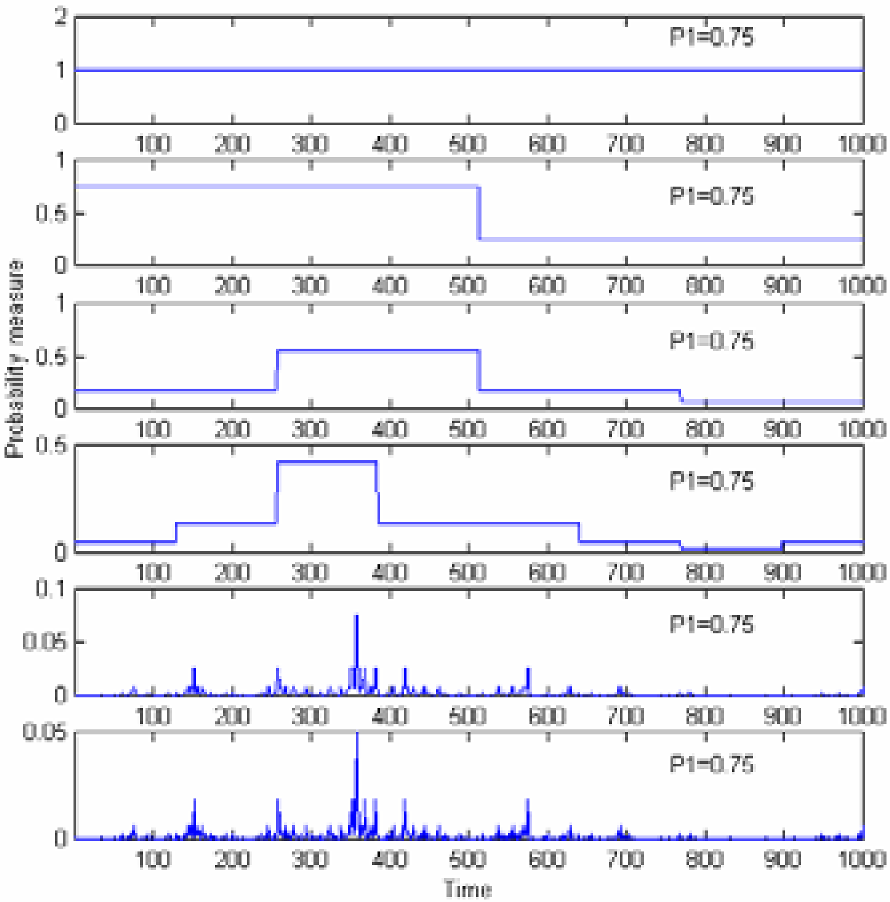


Different fractions (μ and $1-\mu$) of energy ($0.5 \leq \mu < 1$) are transferred, in a conservative way, from the mother eddy to the two daughter eddies.

The fraction μ is constant at each step, and the "side" (left or right) is chosen at random

A general fragmentation process !

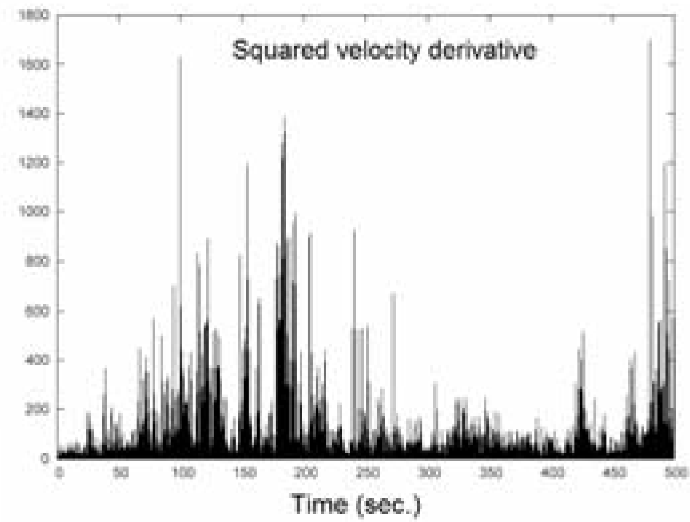
The energy transfer rate in the p-model



A 1D simulation with μ = 0.75

The density of energy transfer rate becomes SINGULAR as the cascade proceeds to small scales.

Small scales show singular regions with high activity and more quiet regions.

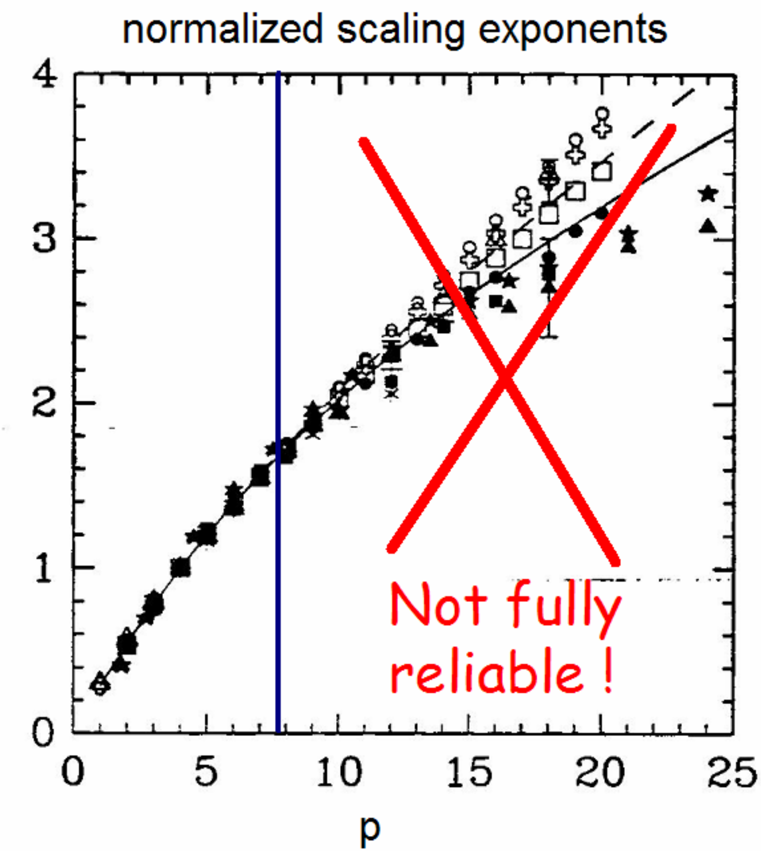


Atmospheric flow

Comparison with velocity in fluid flows

A binomial process → analytical expression for the scaling exponents

$$\varsigma_p = 1 - \log_2\left[\mu^{p/3} + (1-\mu)^{p/3}\right]$$

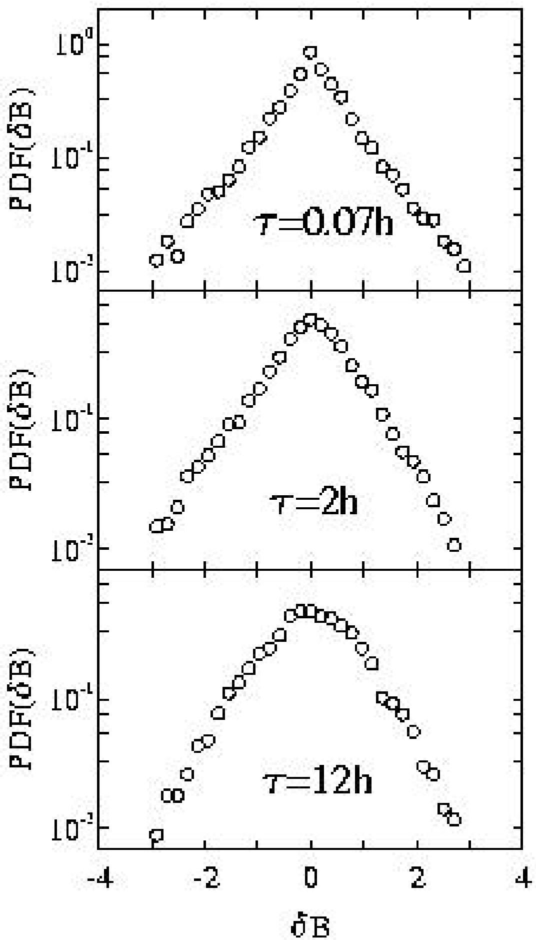


The free parameter can be used to "tune" the intermittency strength.
Best fit on the data μ = 0.7
Note: unfortunately the curve is smooth enough! Different models gives acceptable results.

A collection of data from laboratory fluid flows (black symbols) and solar wind velocity (white symbols).

Differences only for unreliable high order moments, due to different geometry of dissipative structures (see later)

A different signature of intermittency: PDFs of fluctuations depend on the scale



$$\delta B_\tau = \frac{\Delta B_\tau}{\sqrt{\langle \Delta B_\tau^2 \rangle}}$$

Standardized variables at different scales

Self-similarity: the pdfs of normalized fields increments at different scales collapse on the same shape

$$pdf(\delta B_{\lambda r}) = pdf(\delta B_r)$$

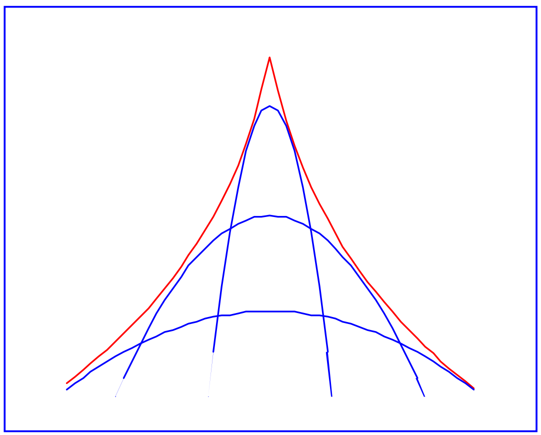
Real turbulence: The dependence of the PDFs of standardized variables from the scale, means that the phenomenon of turbulence **CANNOT** be considered as being **globally self-similar**.

A model for PDF scaling

The departure from self-similarity can be described through a multifractal model that represents the scaling evolution of PDFs

Convolution of gaussians G with different standard deviations σ, according to a distribution L(σ)

$$P_\tau(\Delta\psi) = \int L(\sigma)\; G(\Delta\psi,\sigma)\; d\sigma$$



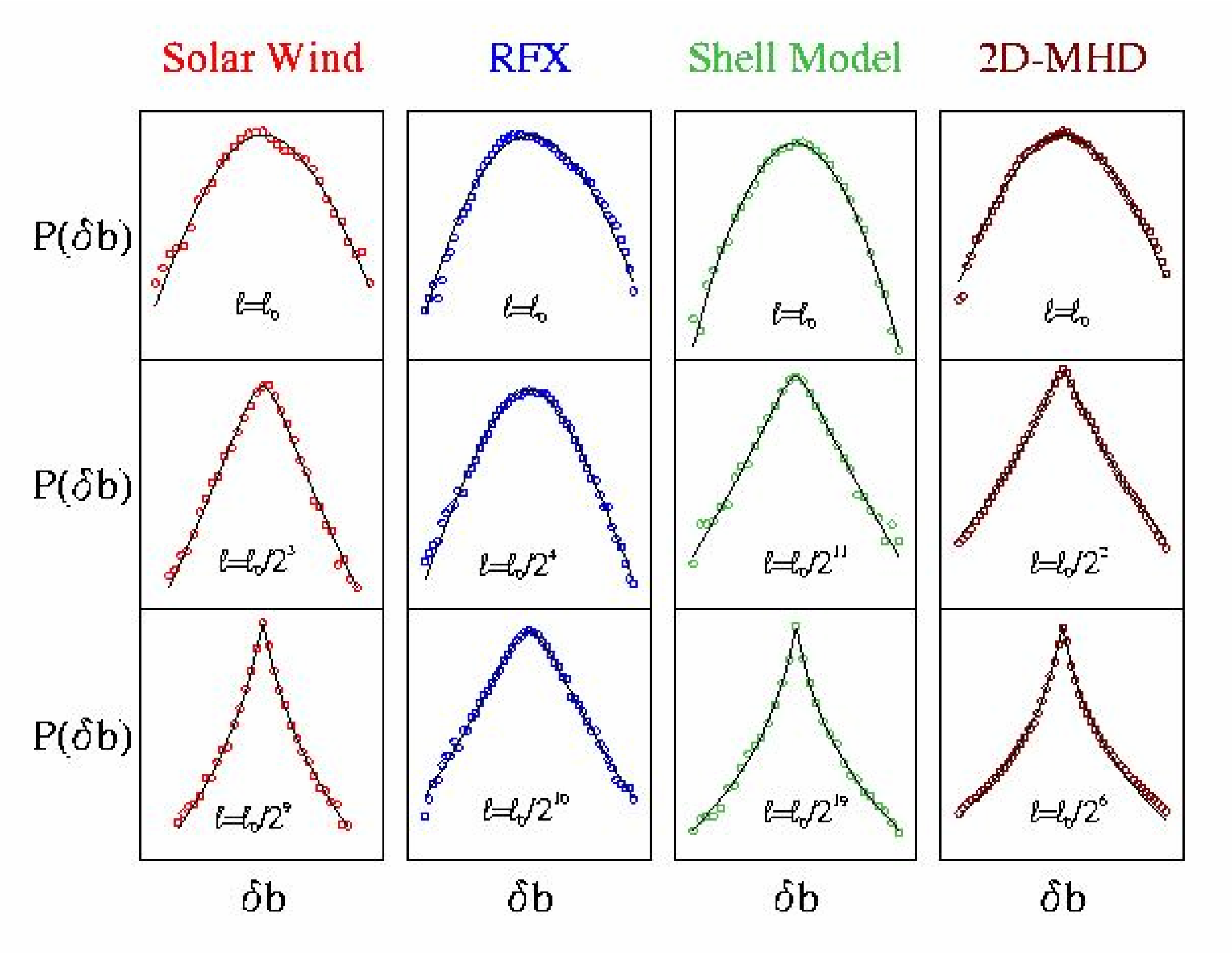
the sum of gaussians of different width σ (blue) yields the “stretched” PDF (red)

For example a log-normal ansatz

$$L(\sigma) = \frac{1}{\sqrt{2\pi\lambda}} \exp\left[-\frac{\ln^2\left(\sigma/\sigma_0\right)}{2\lambda^2}\right]$$

Width (variance) of the Log-normal distribution

The model describes scaling evolution of pdfs



The parameter λ^2 can be used to characterize the scaling of the shape of the PDFs, that is the intermittency of the field!

The parameter λ^2 is found to behave as a *power-law* of the scale

$$\lambda^2(r) \approx r^{-\beta}$$

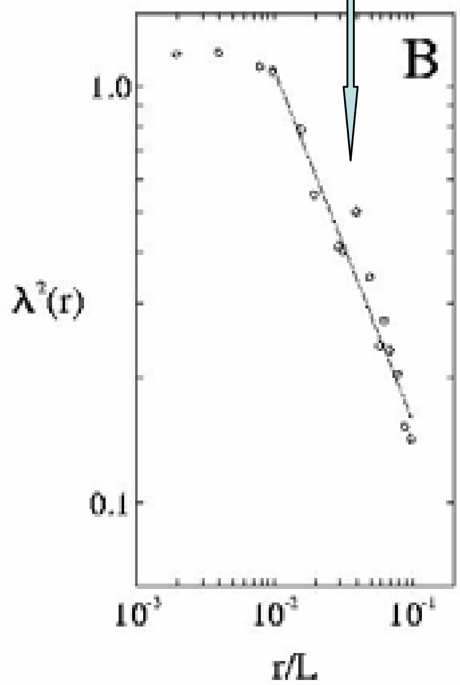
To characterize intermittency, only two parameters are needed, namely:

- λ^2_{max}, the maximum value of the parameter λ^2 within its scaling range, represents the *strength* of intermittency

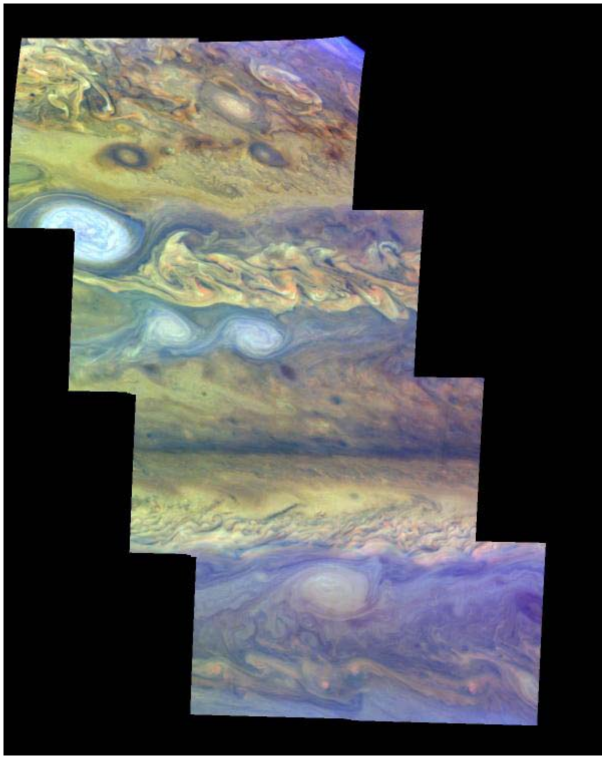
  (the *intermittency level* at the bottom of the energy cascade)

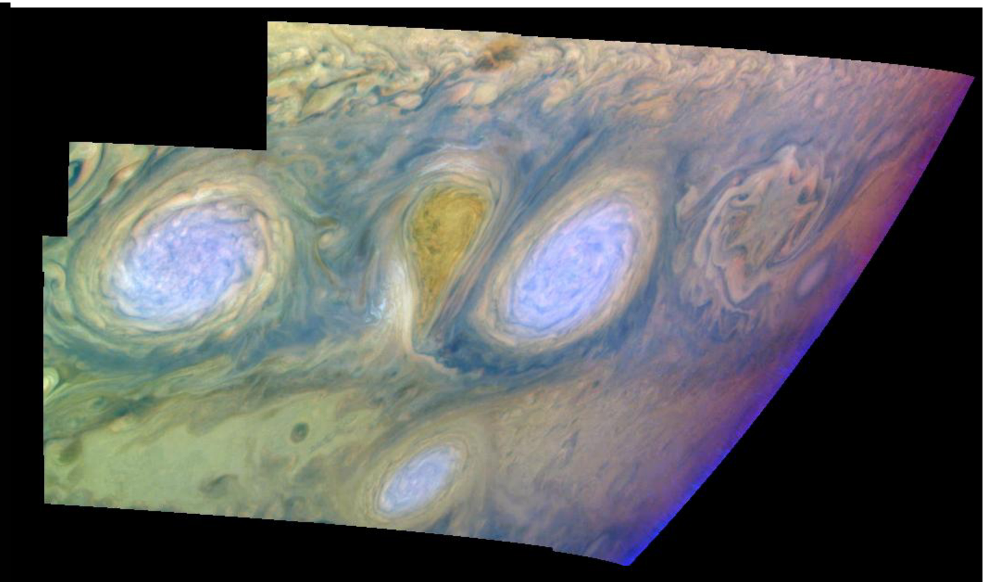
- β, the 'slope' of the power-law, representing the *efficiency* of the non-linear cascade

  (measures *how fast* energy is concentrated on structures at smaller and smaller scales)



Turbulence: “structures” + background ?

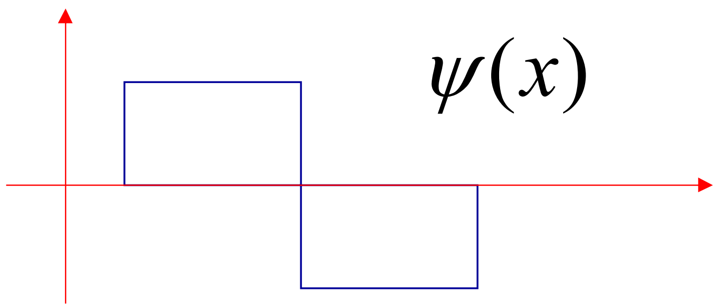




A description of turbulence (since Leonardo!): “coherent” structures present on ALL dynamically interesting scales within a sea of a gaussian background. They contain most of the energy of the flow and play an important dynamical role.

Orthogonal Wavelets decomposition

Let us consider a signal f(x) made by N = 2^m samples, and build up a set of functions starting from a "mother" wavelet



Then we generates from this a set of analysing wavelets by DILATIONS and TRANSLATIONS

Scale Position

$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} w_{ij} \psi_{ij}(x)$$

$$\psi_{ij}(x) = 2^{-j/2} \psi\left(\frac{x - i2^j}{2^j} \right)$$

$$w_{ij} \approx f(x+r) - f(x)$$

$$w_{ij} = \int_{-\infty}^{\infty} f(x) \psi_{ij}(x) dx$$

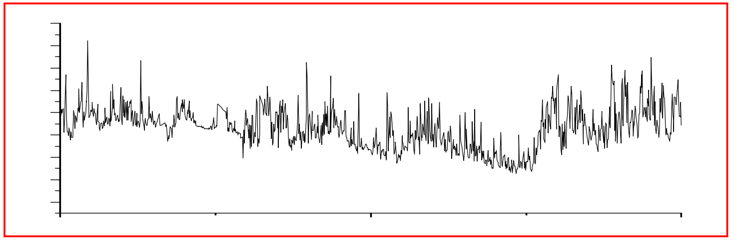
$$\int_{-\infty}^{\infty} \left| f(x) \right|^2 dx = \sum_{ij} \left| w_{ij} \right|^2$$

Local Intermittency Measure

The energy content, at each scale, is not uniformly distributed in space

$$l.i.m. = \frac{\left|w_{ij}\right|^2}{\left\langle \left|w_{ij}\right|^2 \right\rangle_i}$$

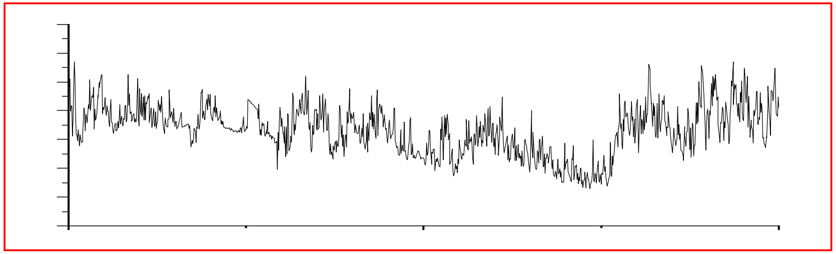
L.i.m. greater than a threshold means that at a given scale and position the energy content is greater than the average at that scale



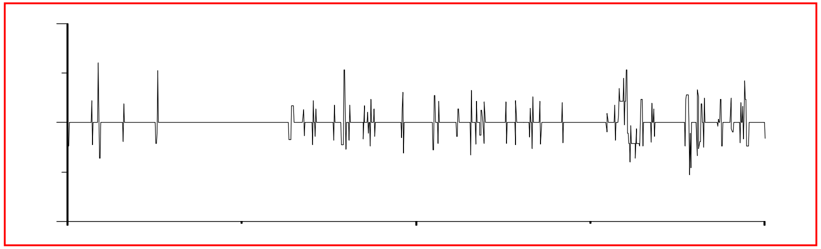
Complete signal

l.i.m. smaller than threshold

l.i.m. larger than threshold

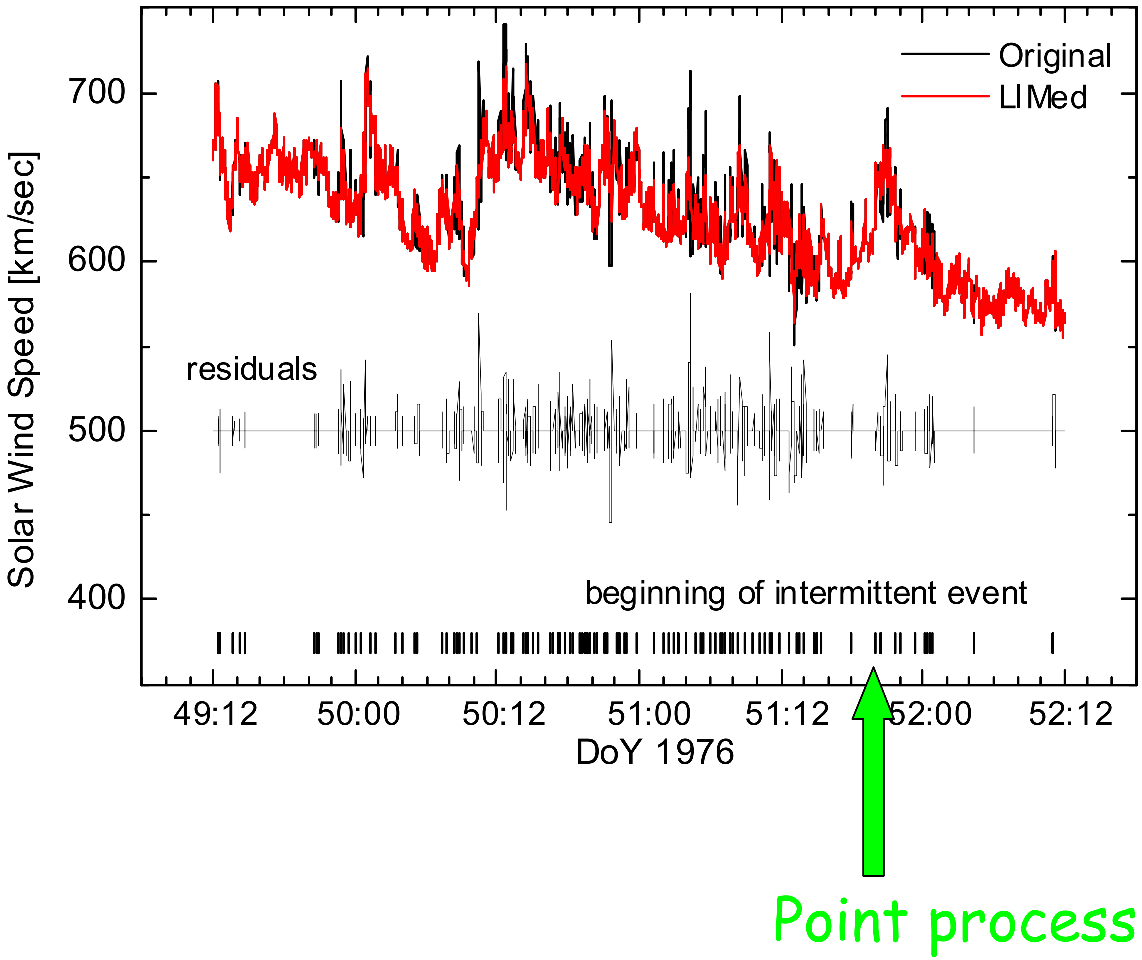


Gaussian background



Structures

In the solar wind

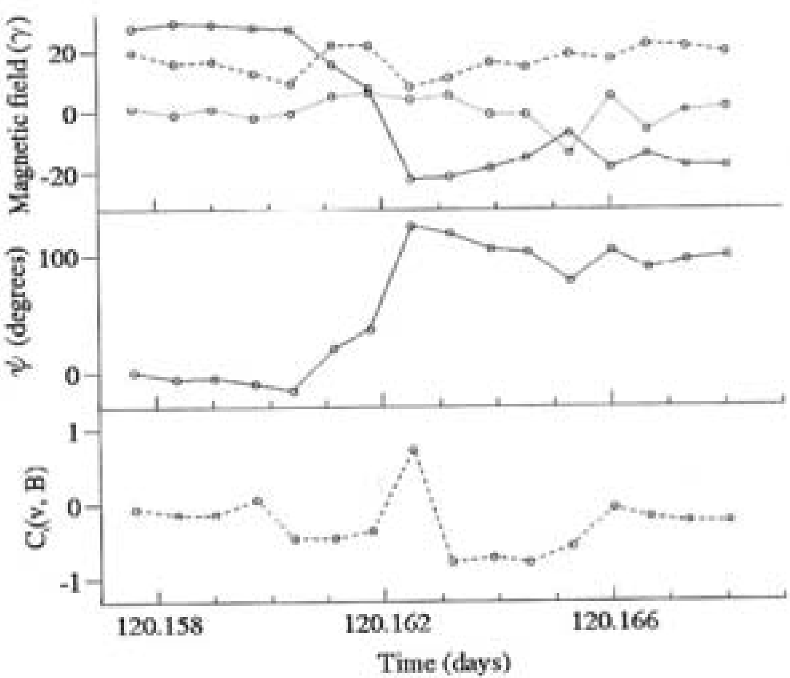


The sequence of intermittent events generates a point process.

Statistical properties of the process gives information on the underlying physics which generated the point process.

What kind of structures in MHD (1)

Minimum variance analysis around isolated structure allows to identify them.

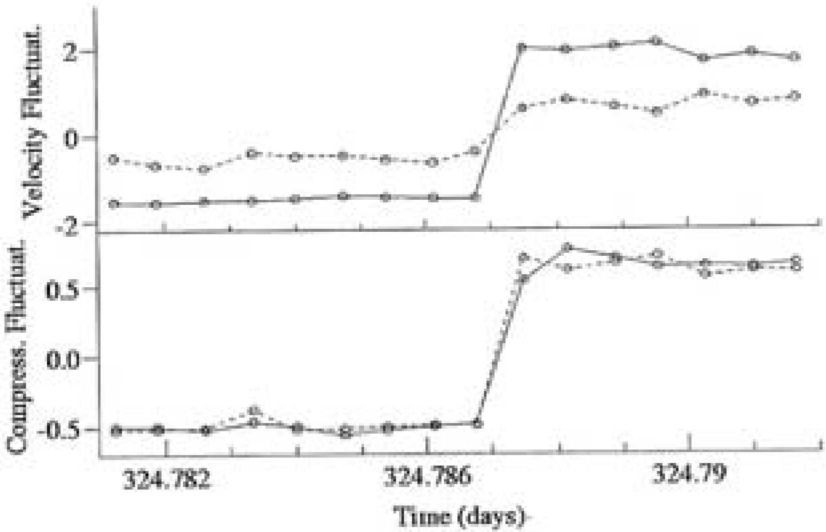


Tangential discontinuity (current sheet) are spontaneously generated **at all scales** inside MHD turbulence by the nonlinear dynamics.

The component of the magnetic field which varies most changes sign, and this component is perpendicular to the average magnetic field (the magnetic field component along the third axis being almost zero). The magnetic field rotates in a plane by an angle of about 120º -130º

What kind of structures in MHD (2)

Minimum variance analysis around isolated structure allows to identify them

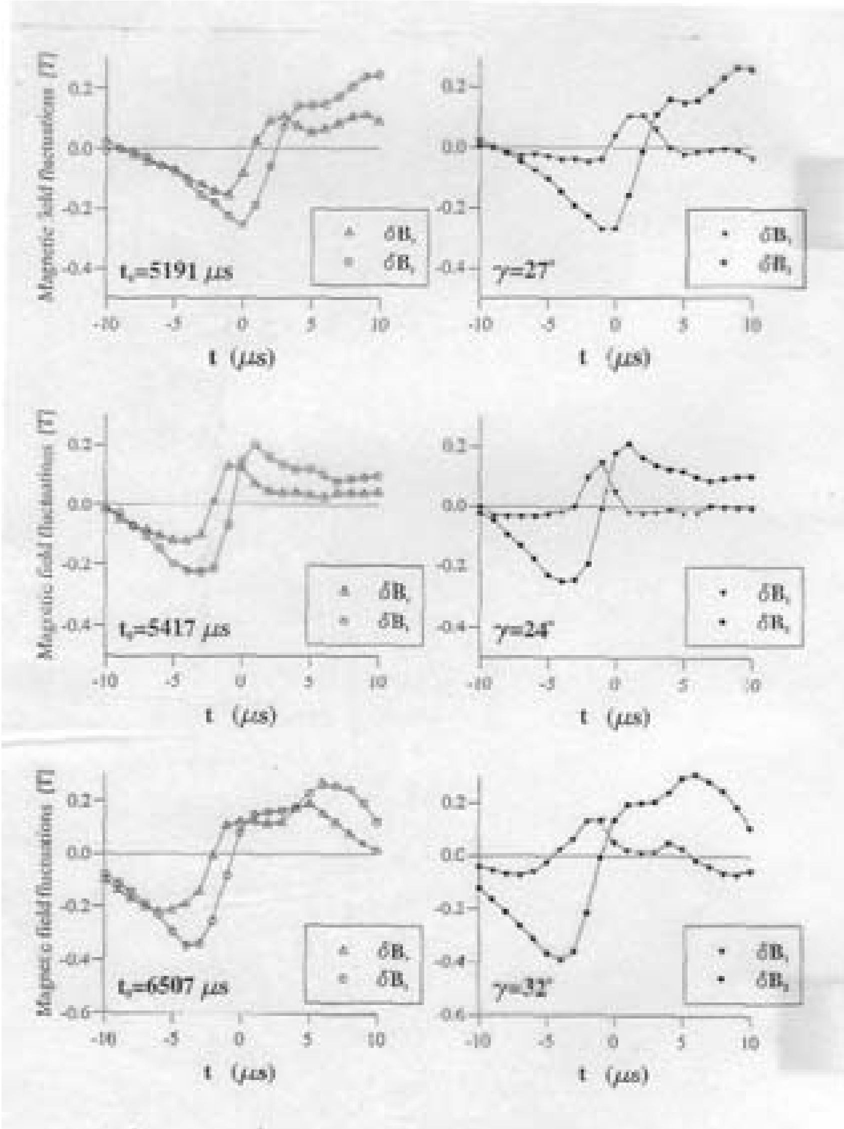


Compressive discontinuities are sometimes observed. These structures can be either parallel shocks or slow-mode (like) wave trains.

Magnetic structures in laboratory plasmas

Edge magnetic turbulence in RFX: current sheets

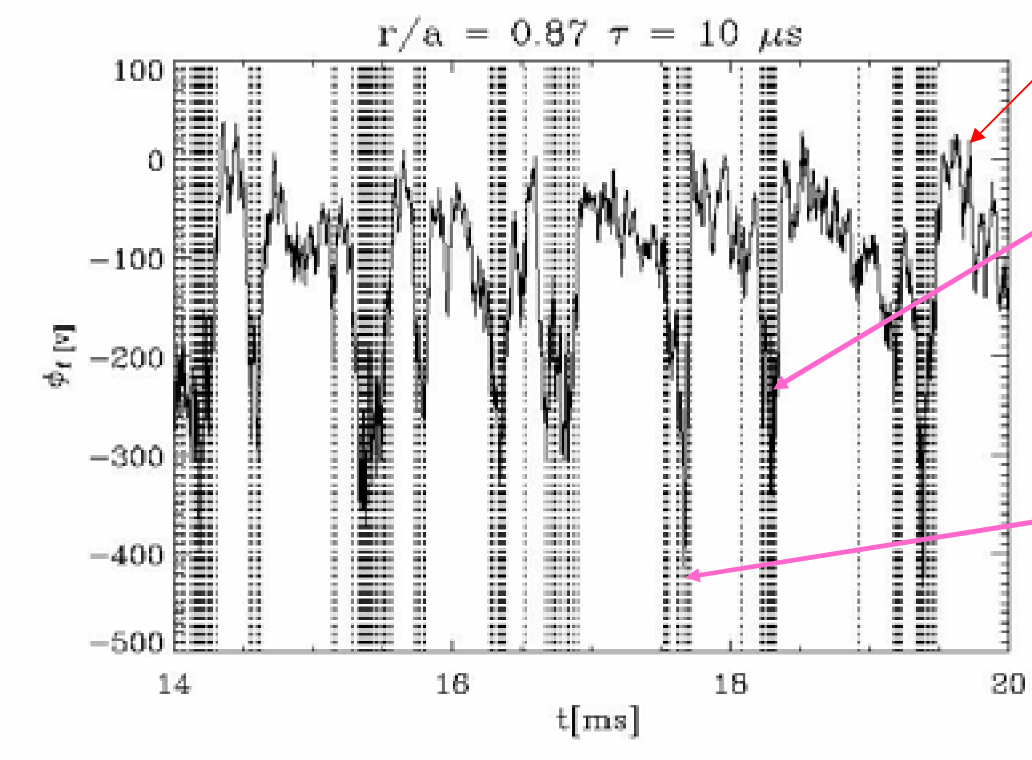
Current sheets are naturally produced as "coherent", intermittent structures by nonlinear interactions



Dynamics of intermittent structures

Relationship between intermittent structures of edge turbulence and disruptions of the plasma columns at the center of RFX

Time evolution of floating potential at edge

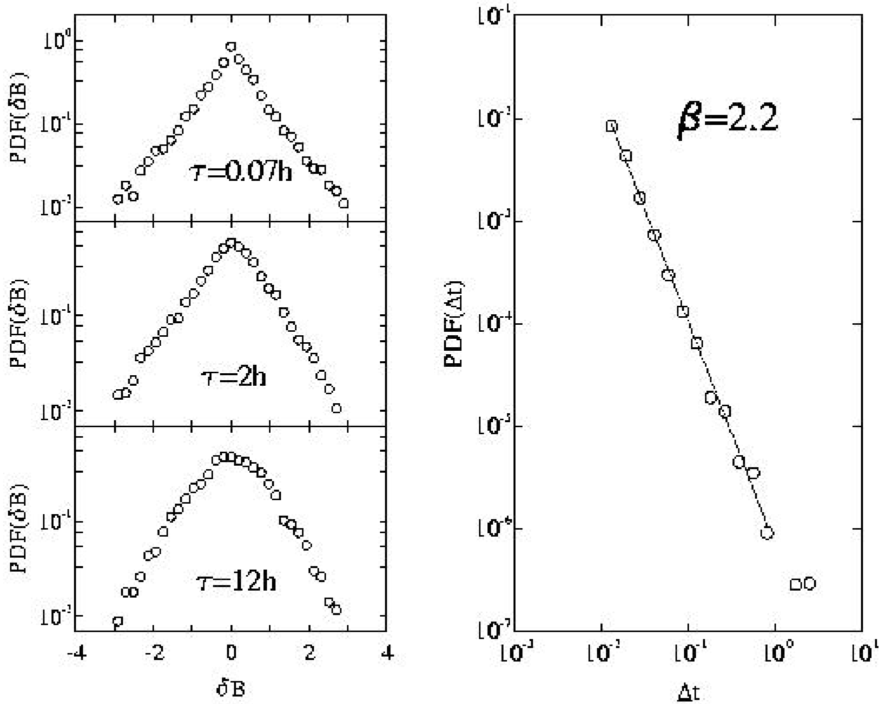


Minima are related to disruptions of the magnetic structure (at the center)

Appearance of intermittent structures in the electrostatic turbulence at the edge of the plasma column (vertical lines)

Yet we don’t have explanation for this!

Playing with point processess!



Solar wind

The times between events (waiting times) are distributed according to a power law

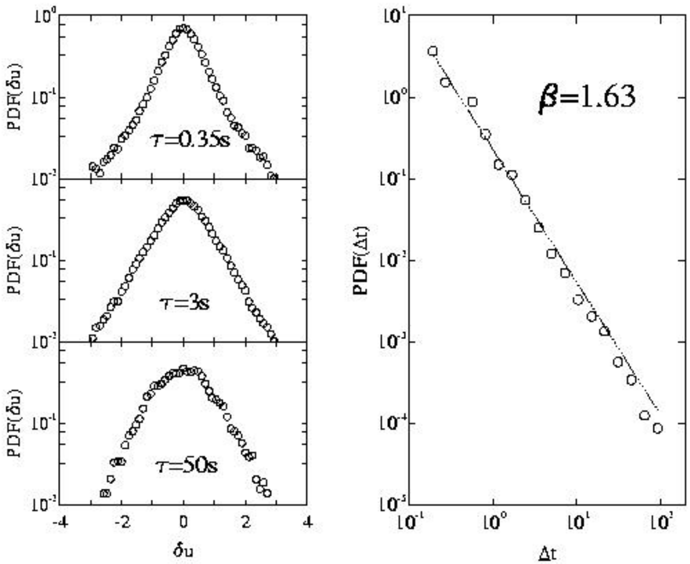
$$Pdf(\Delta t) \sim \Delta t^{-\beta}$$

The turbulent energy cascade generates intermittent "coherent" events at all scales.

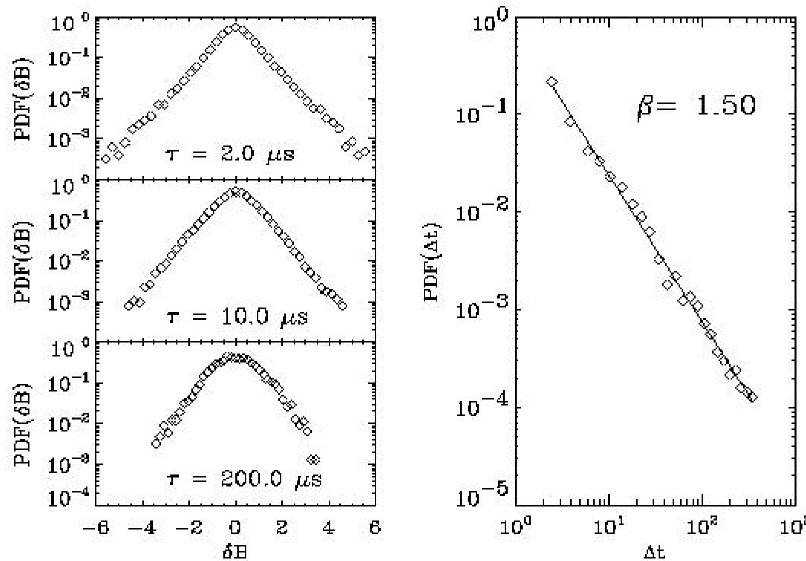
Interesting! the underlying cascade process is NOT POISSONIAN (we found no exponential pdfs), that is the intermittent (more energetic) bursts are NOT INDEPENDENT (memory)

Power law distribution for waiting times

Fluid flow



Laboratory plasma



(Perhaps) all turbulent flows share this characteristic.
Power laws must be reproduced by models for turbulence.

3th question

Where the turbulence of water comes to rest (?)

Dissipation of energy in classical turbulence

When the dissipative time becomes of the order of the nonlinear eddy turnover time, the energy cannot be transferred efficiently. The process becomes dissipative and energy is dissipated.

$$\partial_t u_i + u_\alpha \partial_\alpha u_i = -\partial_i P + \nu \partial_\alpha^2 u_i$$

$$\frac{\partial \mathbf{u}(\mathbf{k},t)}{\partial t} \sim -\nu k^2 \mathbf{u}(\mathbf{k},t)$$

$$\mathbf{u}(\mathbf{k},t) \sim \exp(-\nu k^2 t)$$

Dissipation length

$$\tau_\eta \sim \tau_D \qquad R_\eta = \frac{u_\eta \eta}{\nu} \simeq 1$$

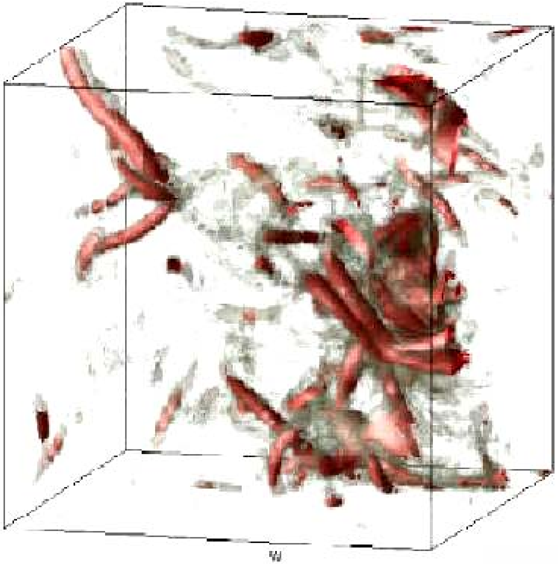
$$\eta \sim \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \qquad \frac{\eta}{L} \sim R^{-3/4}$$

Since $R=10^6$ for the swimmer

$$\begin{aligned} \eta &\simeq 3 \times 10^{-2} mm \\ L_D &\simeq 30\eta \end{aligned}$$

We cannot observe structures on scales lesser than 1 mm

Dissipation through isolated bursts: finite-time singularities



Numerical simulation

Dissipative structures are very localized both in space and time (intermittency in the dissipative domain). **Energy is dissipated through isolated bursts.**
This process can be viewed as a generation of finite-time singularities:

Eddy-turnover time: lifetime of eddies

$$\tau_\ell \sim \frac{\ell}{u_\ell} \sim \epsilon^{-1/3}\ell^{2/3}$$

$$T_{tot} \sim \sum_\ell \ell^{2/3}$$

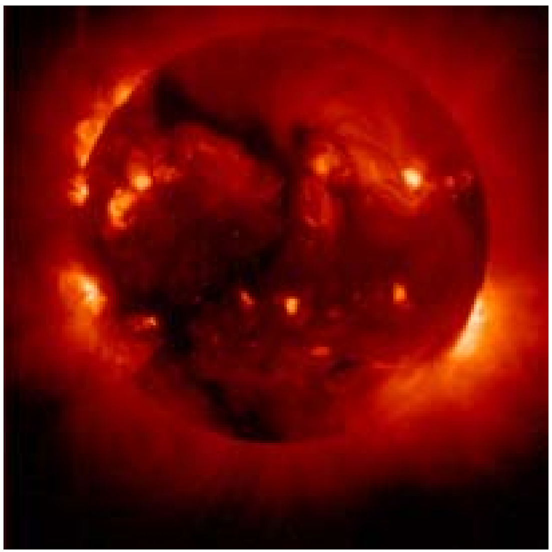
The sum of eddy-turnover times CONVERGES as the scale length tends to zero.

The energy is transferred towards structures of ZERO length in a FINITE time, this generates singularities in the dissipative domain.

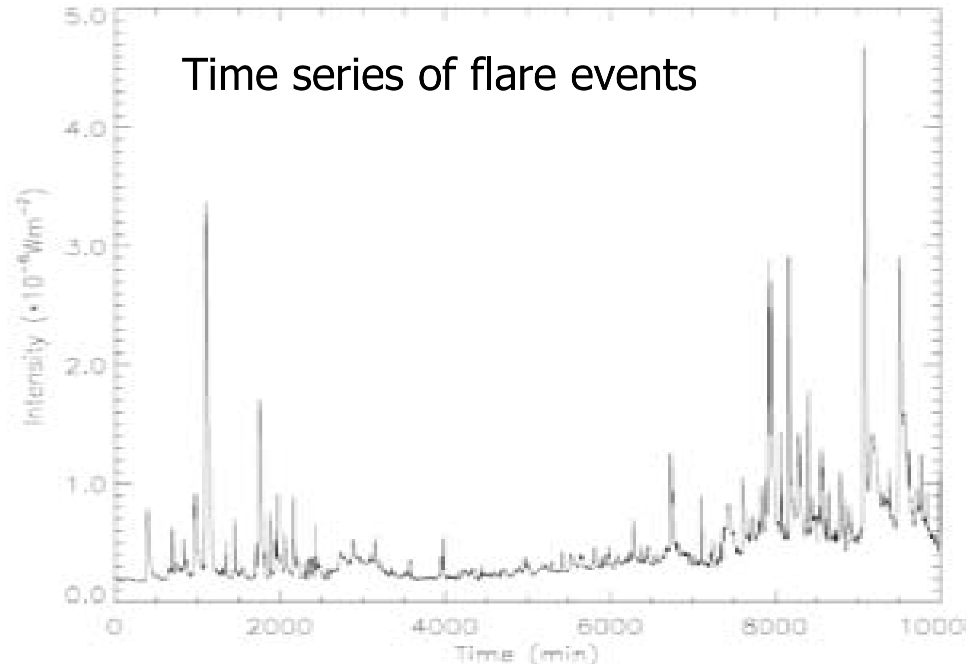
What happens in real plasma turbulence?

Example of Solar flares: impulsive annihilation of magnetic energy at spontaneously generated current sheets in a turbulence inside the solar corona (?)

(Boffetta, Carbone, Giuliani, Veltri, Vulpiani, Phys. Rev. Lett. 1999)

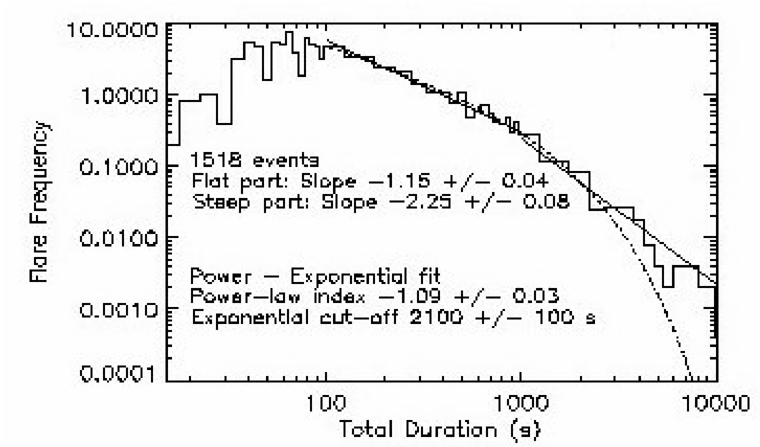


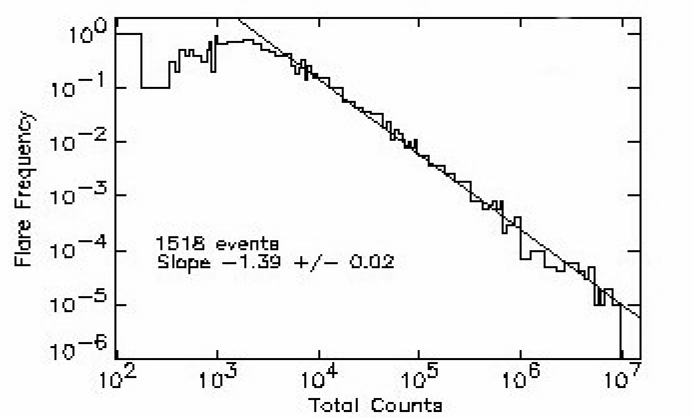
Hard X-ray (> 20 keV):
Intermittent spikes, duration 1-2 s,
$E_{max} \sim 10^{27}$ erg
Numerous smaller spikes down to 10^{24} erg (detection limit)

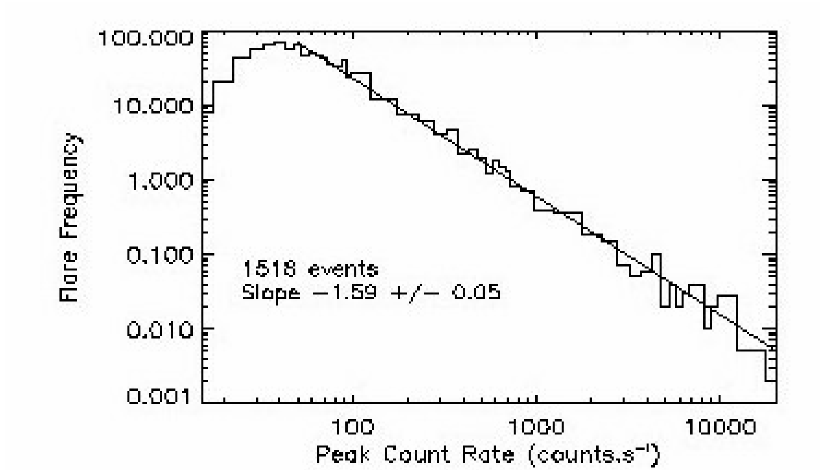


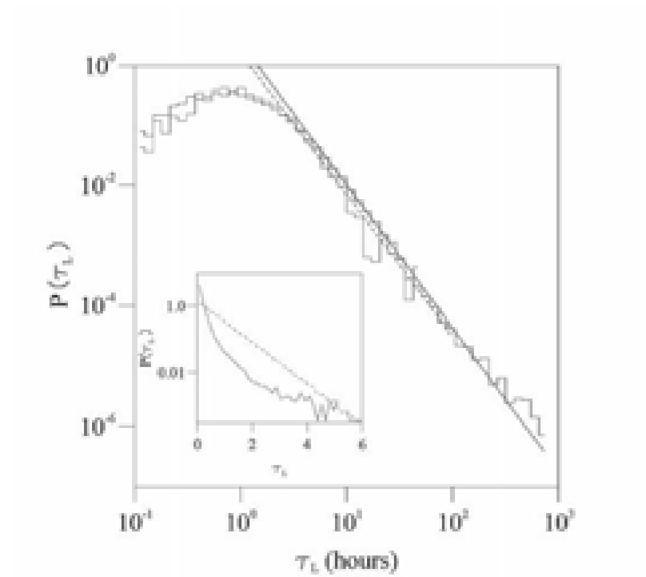
Parker's X-ray corona: superposition of a very large number of flares (NANOFLARES)

Power law statistics of bursts





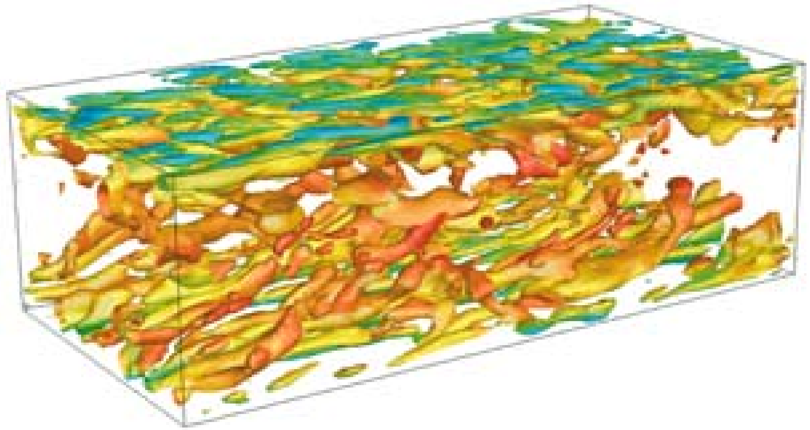




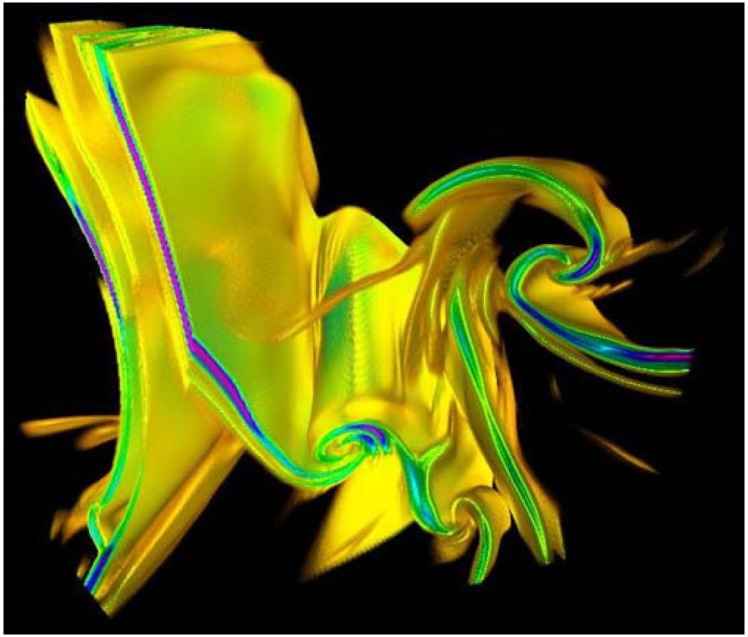
Total energy, separation times, peak energy and (more or less!) lifetime of individual bursts seems to be distributed according to power laws.

Geometry of dissipative bursts

Intermittent dissipative structures:
Filaments in usual fluid flows, sheets in MHD flows

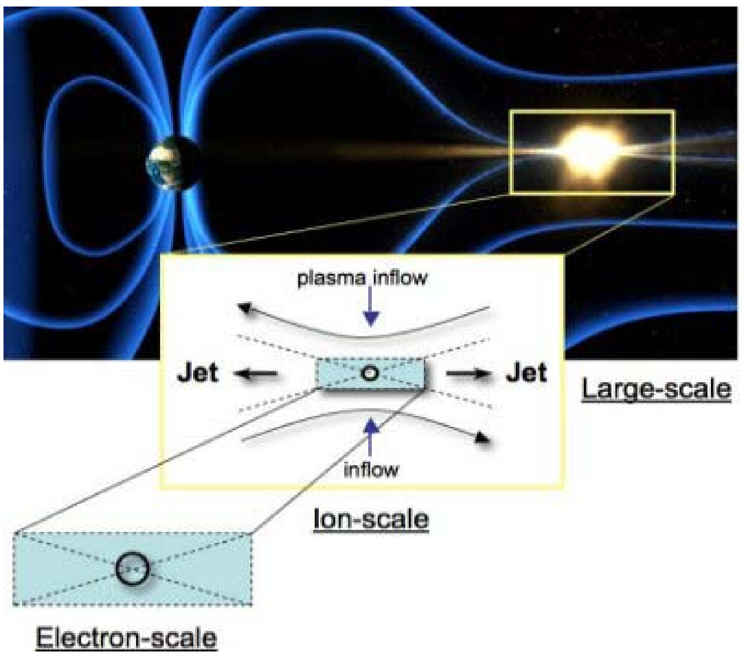


Dissipative structures near the wall



Current sheet

Current sheets have been observed in space plasmas (Cluster spacecrafts)



Current sheets are very interesting, because they are the place for the occurrence of a lot of physical processes: magnetic reconnection, particle acceleration, bursty magnetic energy annihilation, etc.

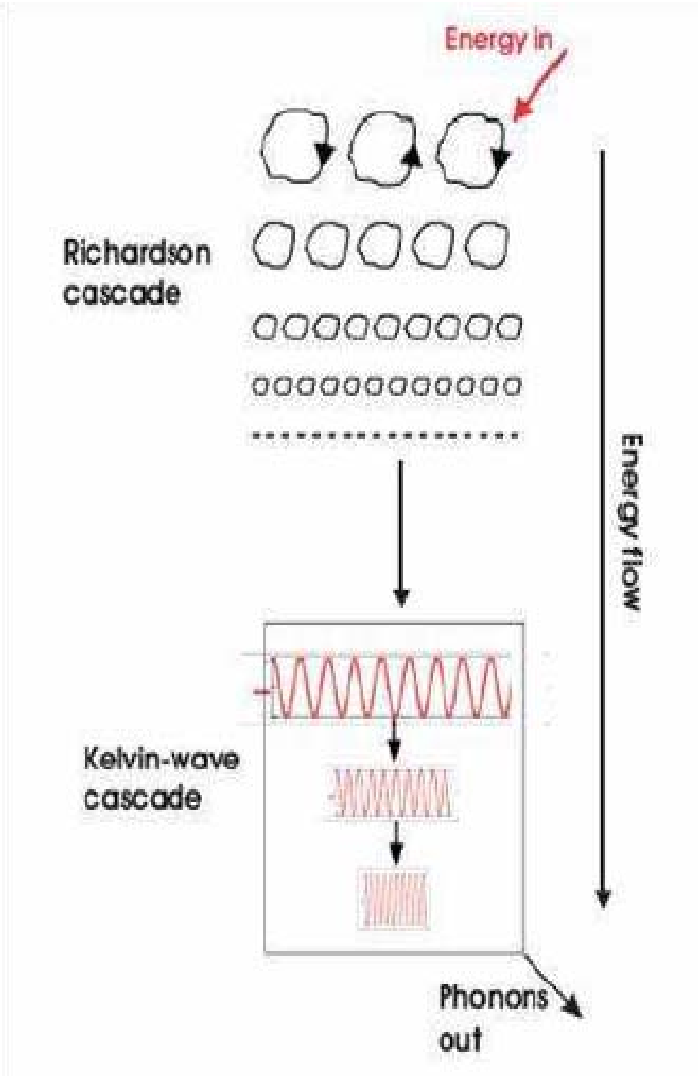
While large-scales in solar wind can be described (more or less) within a fluid approach, dissipation is much more (perhaps completely) different.

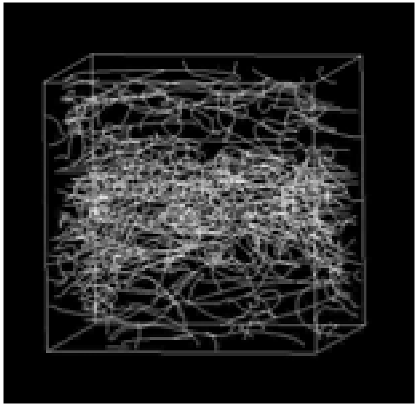
Mean-free-path $\rightarrow$ $\lambda = 10^{13}$ cm
(3 times Sun-Earth distance)

Spacecrafts observe a collisionless fluid !

The “dissipative” range of turbulence in solar wind should be similar to quantum turbulence (nonviscous superfluid phase of ^{3}He).

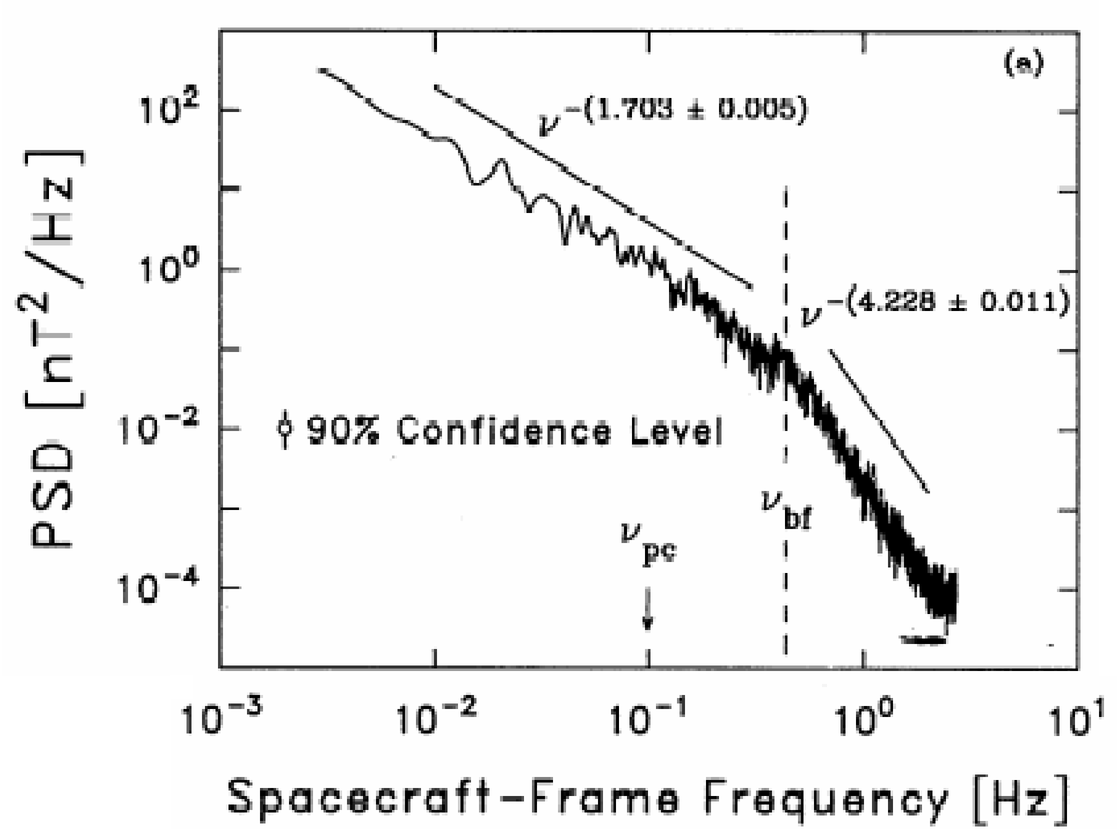
Superfluid turbulence





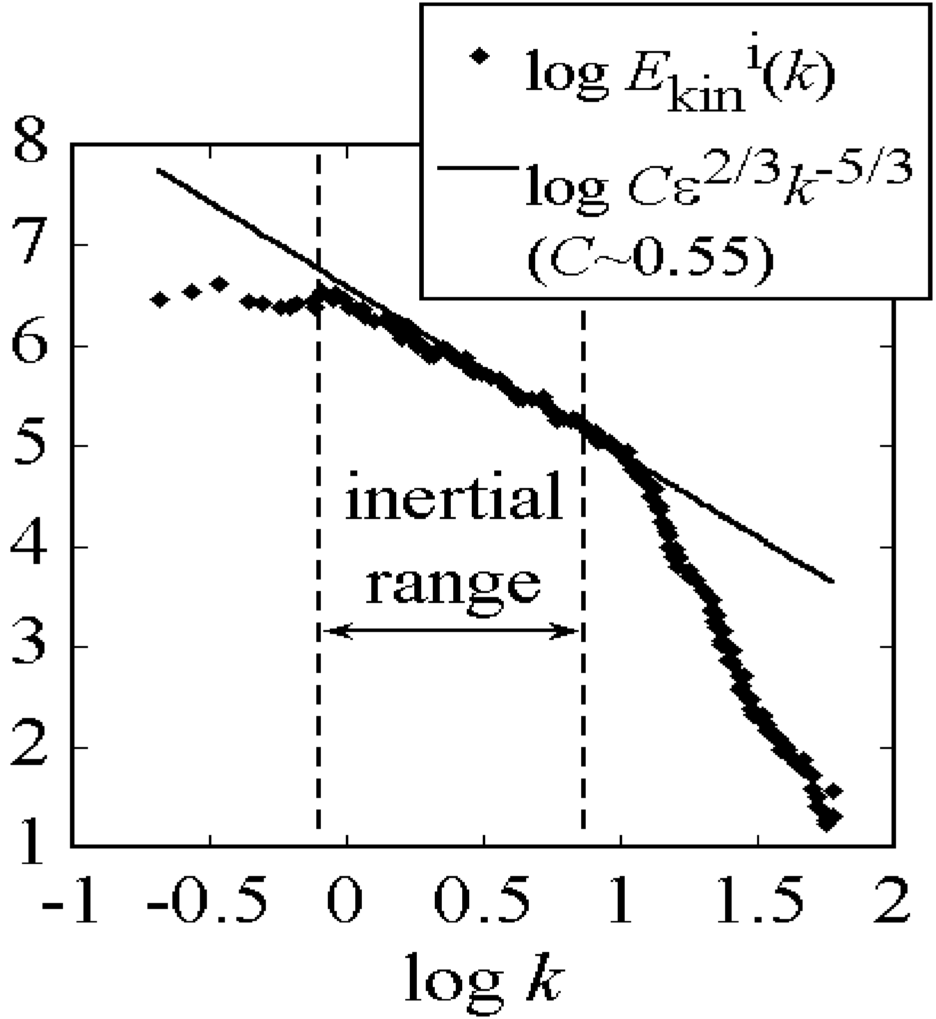
- Energy enters, creating vortices on large scales.
- Reconnections $\Rightarrow$ flow of energy to smaller scales.
- Cascade cannot end in viscous dissipation!
- Instead, cascade of Kelvin waves to higher k.
- Eventually, direct phonon generation.

Do we observe a “dissipative range” in the solar wind turbulence?



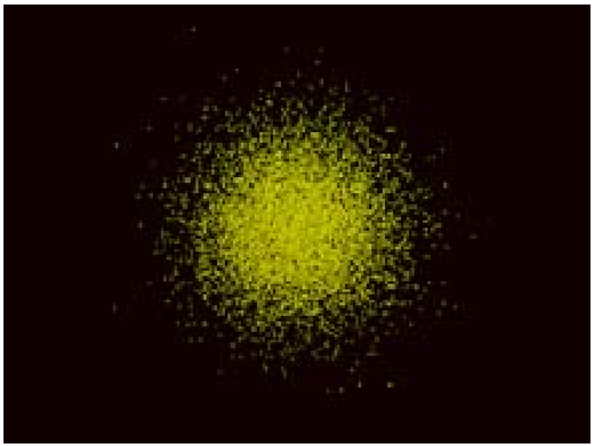
- spectra ~ 2 power laws: in origin attributed to “Inertial range” & “Dissipation range”
- break in the vicinity of the proton cyclotron frequency
- on average $f^{-5/3}$ in the low-frequency range
- on average $f^{-7/3}$ in the high-frequency range

Energy power spectrum in superfluid turbulence

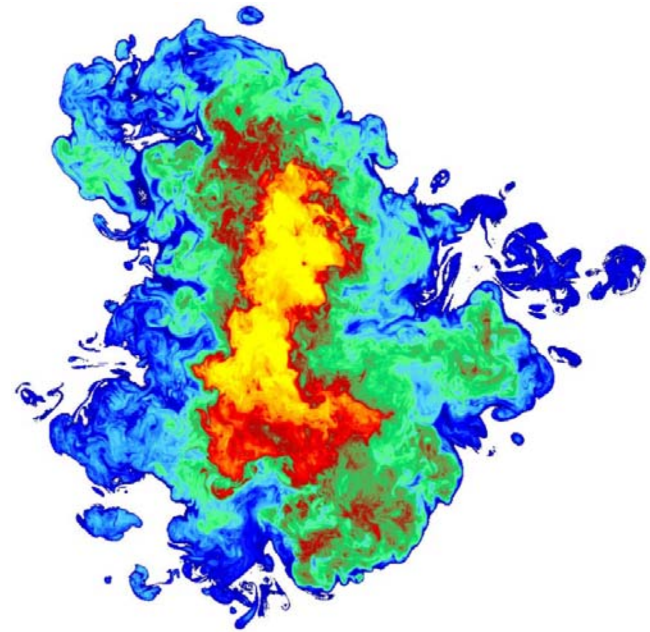


Why investigating turbulence? Increasing transport coefficients

Brownian diffusion



Turbulent diffusion



$$R \approx \sqrt{t}$$

Average distance of pairs

$$R \approx \sqrt{t^3}$$

1) Cuba-libre: few h
2) Heating a normal room: 347 h

Il trascinamento (drag) dovuto alla turbolenza ha un impatto terribilmente negativo per esempio nel moto delle automobili o nel trasporto di fluidi nelle condutture (petrolio, olio combustibile, etc..)

Parametri tipici dell'oggetto:
Potenza: $P = 45$ kW
Dimensioni: $L = 2$ m

Parametri tipici del fluido:
Densità: $\rho = 1.2$ kg/m^3
Viscosità: $\upsilon = 2 \times 10^{-5}$ Pa s

Velocità che dovrebbe avere l'auto SENZA turbolenza

Forza di Stokes agente: $f = 6\pi\upsilon LV$
Potenza dissipata: $P = fV$

$$\boxed{V = \sqrt{\frac{P}{6\pi\upsilon L}} = 25000\ \mathrm{km/h}}$$

In condizioni stazionarie

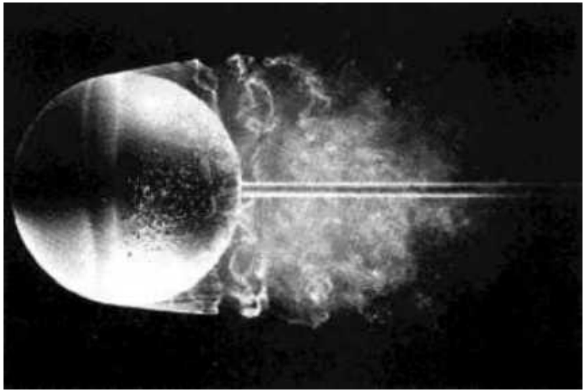
Velocità che l'auto ha per il fatto che esiste il drag turbolento

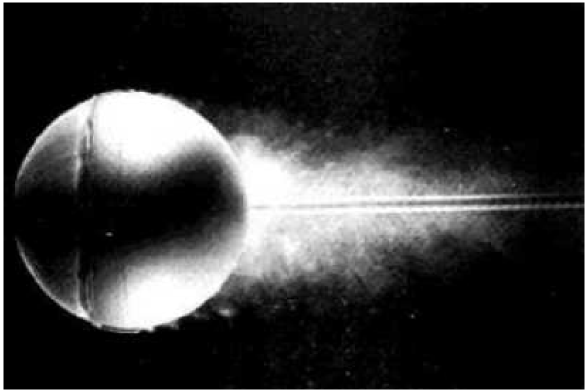
Tempo di generazione del vortice: $t = L/V$
Energia del vortice: $E = \frac{1}{2}\rho L^3 V^2$
Potenza dissipata dalla turbolenza: $P = E/t$

$$\boxed{V = \sqrt[3]{\frac{2P}{\rho L^2}} = 95\ \mathrm{km/h}}$$

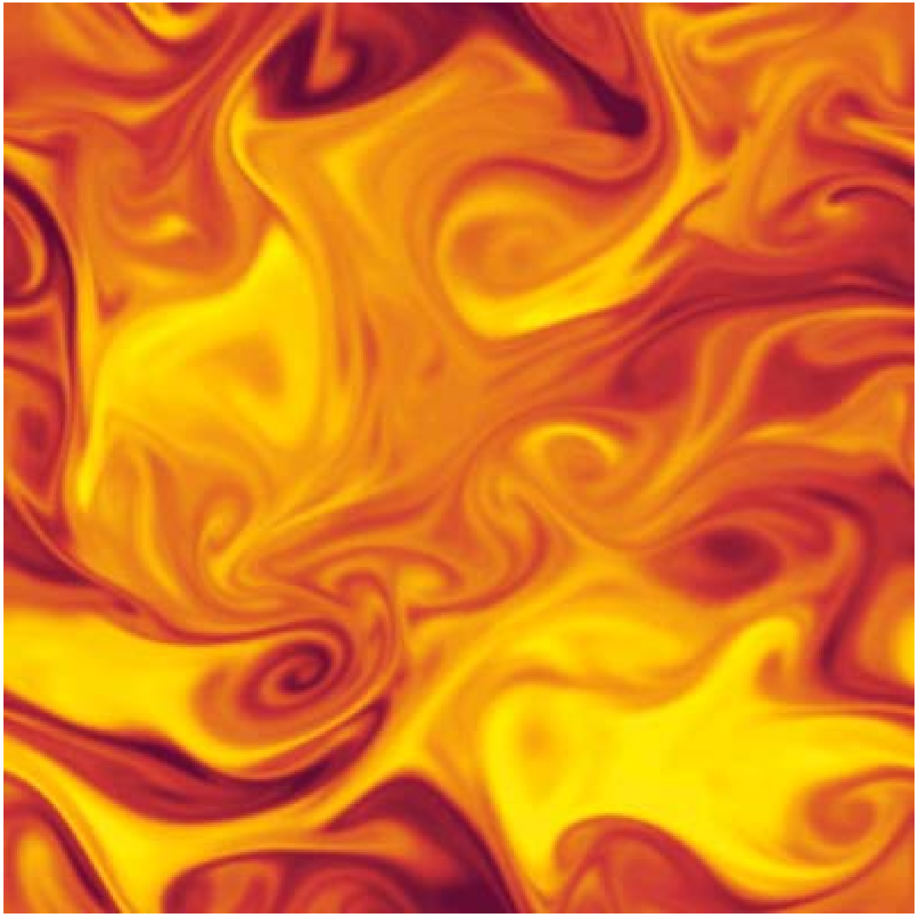
Nota: non dipende dalla viscosità !

Drag reduction: turbulence is investigated also for practical purposes





Modeling turbulence



All features are (more or less) reproduced by numerical simulations. Why models for turbulence?

In the limit of high R, assuming a Kolmogorov spectrum $E(k) \sim k^{-5/3}$ dissipation takes place at scale:

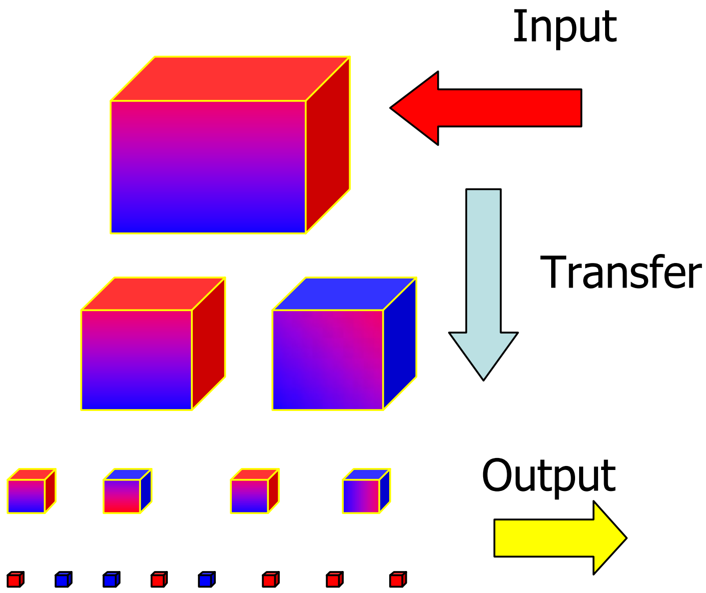
$$l_D \approx L R^{-3/4}$$

the # of equations to be solved is proportional to

$$N \approx (L / l_D)^3 \approx R^{9/4}$$

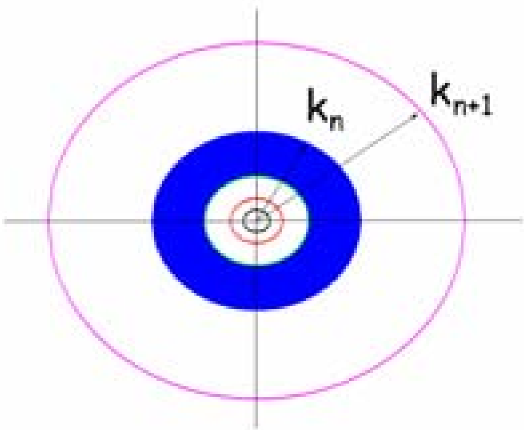
For space plasmas: $R \sim 10^8 - 10^{15}$

Typical values at present reached by high resolution direct simulations
$R \sim 10^3 - 10^5$



At these values it is not possible to have an inertial range extended for more than one decade. No possibility to verify asymptotic scaling laws, statistics...

How to build up shell models (1): Introduce a logarithmic spacing of wave vectors space (shells)



$$k_n = k_0 \lambda^n$$

$$n = 1, 2, \ldots, N$$

The intershell ratio in general is set equal to λ = 2.

In this way we can investigate properties of turbulence at very high Reynolds numbers.

We are not interested in the dynamics of each wave vector mode of Fourier expansion, rather in the gross properties of dynamics at small scales.

How to build up shell models (2): Assign to each shell ONLY one (in fluids) or two (in MHD) dynamical variables;

$$z_n^{\pm}(t) = u_n(t) \pm b_n(t)$$

In this way we ruled out the possibility to investigate BOTH spatial and temporal properties of turbulence. We loose geometrical effects due to different orientation of wave vectors

These variables take into account the averaged effects of velocity modes between k_n and k_{n+1}, that is fluctuations across eddies at the scale $l_n \sim k_n^{-1}$

Shell variables at a given scale play the role of increments at a given separation

$$u_n(t) \rightarrow u(x+\ell_n) - u(x)$$

$$\left\langle u_n^p \right\rangle \rightarrow \left\langle \left[u(x+\ell) - u(x)\right]^p \right\rangle$$

How to build up shell models (3): Write a nonlinear model with quadratic couplings and fix as more as possible the coupling coefficients

$$\frac{\partial z_\alpha^\pm(\mathbf{k},t)}{\partial t} = M_{\alpha\beta\gamma}(\mathbf{k})\sum_p z_\beta^\pm(\mathbf{p},t) z_\gamma^\mp(\mathbf{k}-\mathbf{p},t) + \upsilon k^2 z_\alpha^\pm(\mathbf{k},t) + f_\alpha^\pm(\mathbf{k},t)$$

$$M_{\alpha\beta\gamma}(\mathbf{k}) = -ik_\beta\left(\delta_{\alpha\gamma} - \frac{k_\alpha k_\gamma}{k^2}\right)$$

MHD equations in Fourier space

$$\frac{dz_n^\pm(t)}{dt} = ik_n \sum_{i,j=\pm 2,\pm 1} M_{i,j} z_{n+i}^\pm(t) z_{n+j}^\mp(t) + \upsilon k_n^2 z_n^\pm(t) + f_n^\pm$$

Interactions between nearest and next nearest shells

Invariants of the dynamics in absence of dissipation and forcing:

1) total energy
2) cross-helicity
3) magnetic helicity

$$E(t) = \int \left[u^2 + b^2\right] dx$$

$$H_c(t) = \int \left(\vec{u}\cdot\vec{b}\right) dx$$

$$H(t) = \int \left[\vec{A}\cdot\vec{\nabla}\times\vec{A}\right] dx$$

3D

$$\boxed{H(t) = \int \left|A\right|^2 dx}$$

2D

GOY shell model

$$\frac{du_n}{dt} = ik_n\left[(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - \frac{\delta}{2}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \frac{1-\delta}{4}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})\right]^*$$

$$\frac{db_n}{dt} = ik_n\left[(1-\delta-\delta_m)(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) + \frac{\delta_m}{2}(u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + \frac{1-\delta_m}{4}(u_{n-2}b_{n-1} - u_{n-2}b_{n-1})\right]^*$$

Conserved quantities

$$E = \sum_n |u_n|^2 + |b_n|^2$$

$$H_c = 2\,\mathrm{Re}\left(\sum_n u_n b_n^*\right)$$

The model conserves also a “surrogate” of magnetic helicity

There is the possibility to introduce “2D” and “3D” shell models.

$$\boxed{H = \sum_n \frac{|b_n|^2}{k_n^2} \qquad \delta = 5/4; \quad \delta_m = -1/3}$$

H positive definite: 2D case

$$\boxed{H = \sum_n (-1)^n \frac{|b_n|^2}{k_n} \qquad \delta = 1/2; \quad \delta_m = 1/3}$$

H non positive definite: 3D case

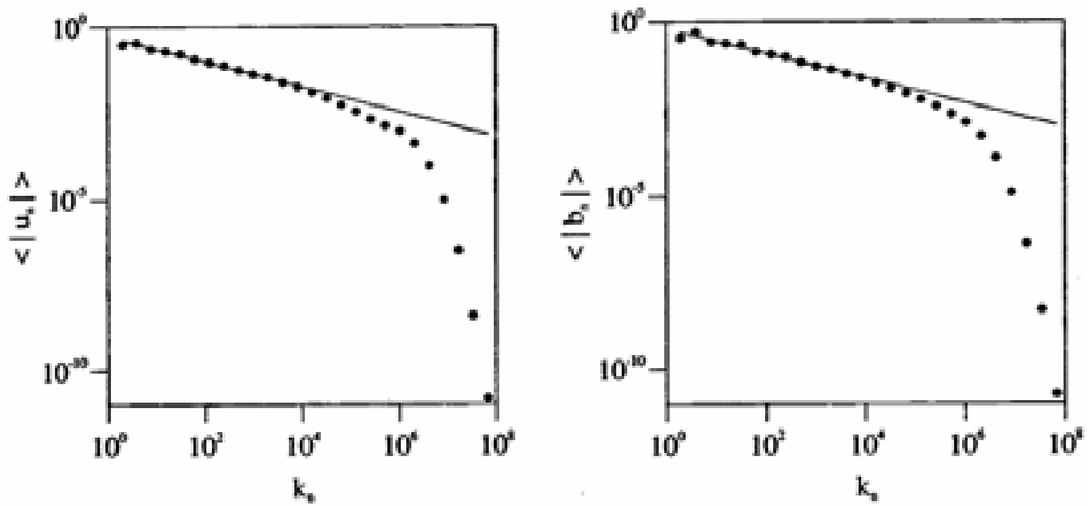
Spectral properties of the 3D model:

Numerical simulations with: N = 26 shells; viscosity = 0.5 · 10^{-9}

We use a Langevin equation for the external forcing term acting only on the velocity field, with a correlation time τ (eddy-turnover time)

$$\frac{df}{dt} = -\frac{f}{\tau} + \mu(t)$$

$$\langle \mu(t)\mu(t') \rangle \approx \delta(t-t')$$

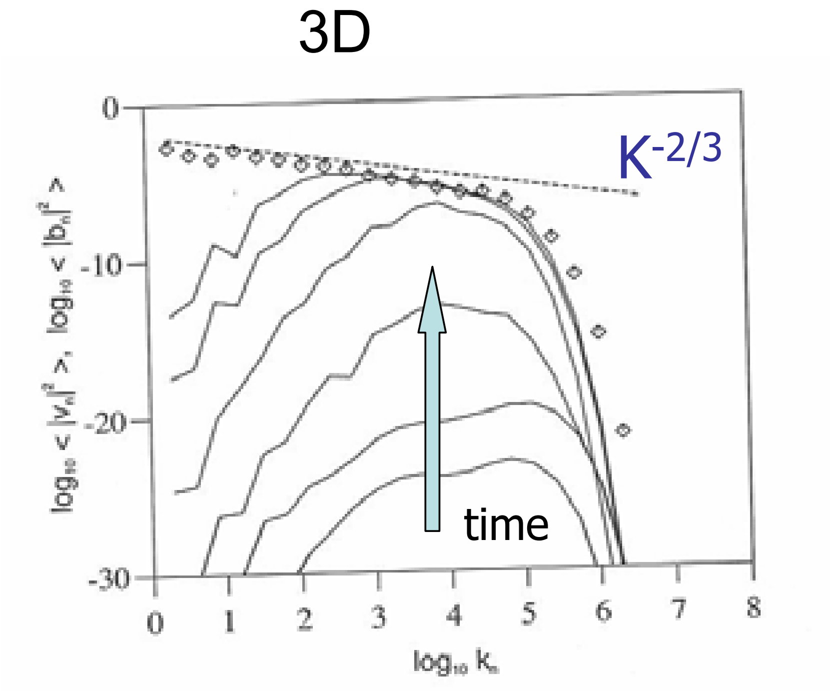


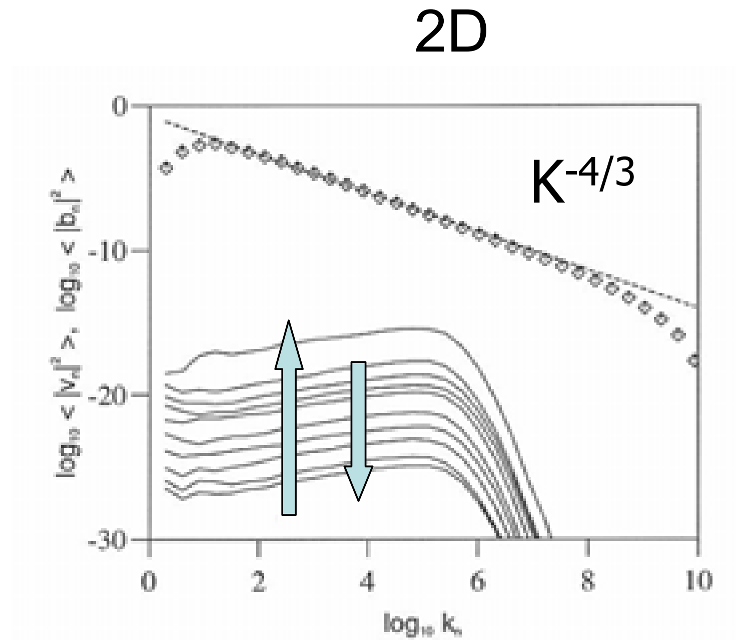
$$\langle u_n^2 \rangle \approx E(k_n)k_n \approx k_n^{-2/3}$$

Kolmogorov spectrum is a fixed point of the system.
Inertial and dissipative ranges + intermediate range well visible in shell models

Properties of 2D and 3D model: dynamo and anti-dynamo action

Time evolution of magnetic energy

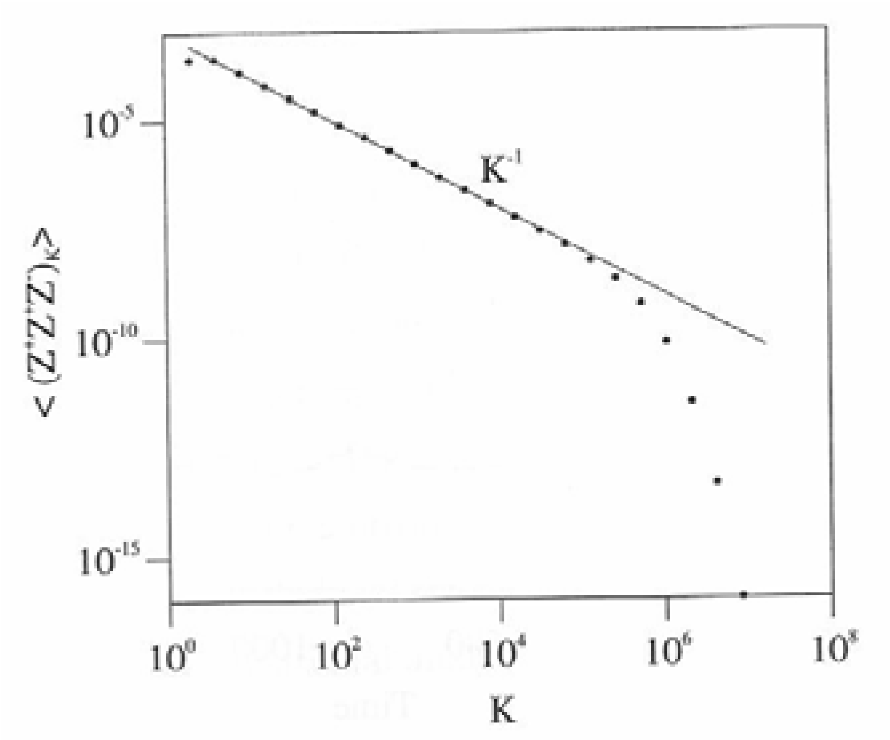




Starting from a seed, the magnetic energy increases towards a kind of equipartition with kinetic energy.

The 2D model shows a kind of "anti-dynamo" action:
a seed of magnetic field cannot increase.

An exact relation for shell model



The analogous of the Yaglom law within the shell model

$$\boxed{\left\langle Y_n^{\pm}(t) \right\rangle = -\frac{4}{3}\varepsilon^{\pm} k_n^{-1}}$$

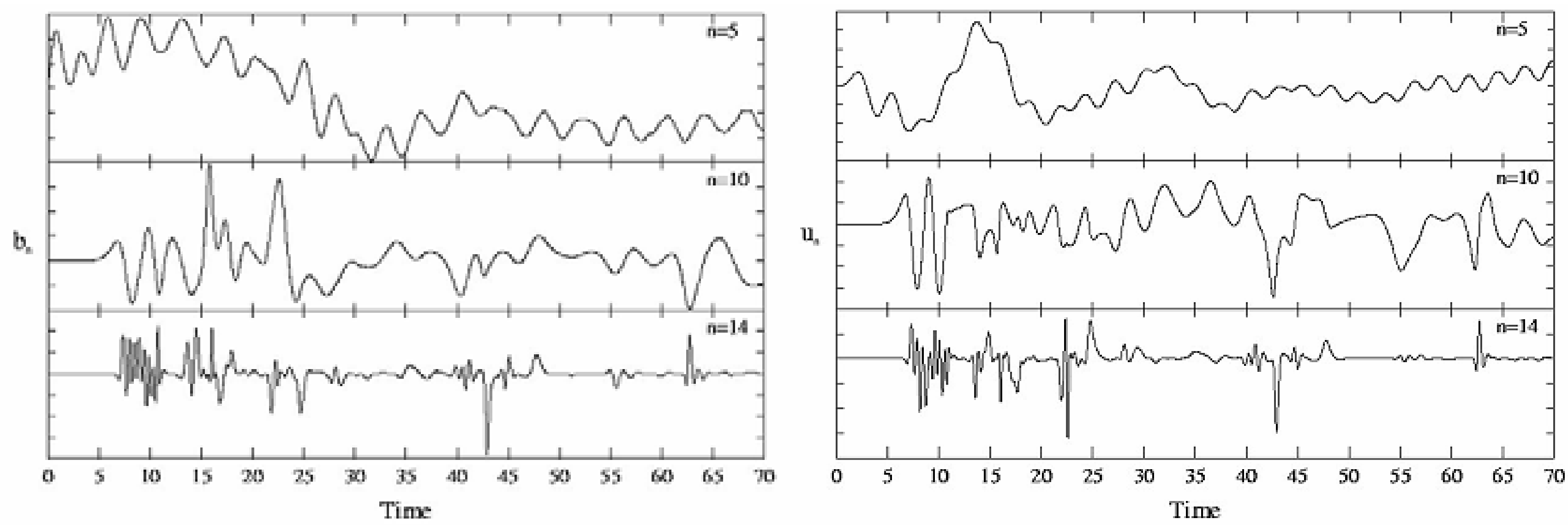
⇓

The 3D shell model reproduces the energy cascade of turbulence

a) The negative sign means that an energy cascade exists;
b) Non-gaussian features of fluctuations

Time intermittency

Time evolution of shell variables at three different shells.



Fields at n = 5 look more like gaussians, fields at n = 14 are made by pulses
→ **time intermittency** (yet poorly understood)

A shell model possess pulse solution of width $O(k_n^{h-1})$

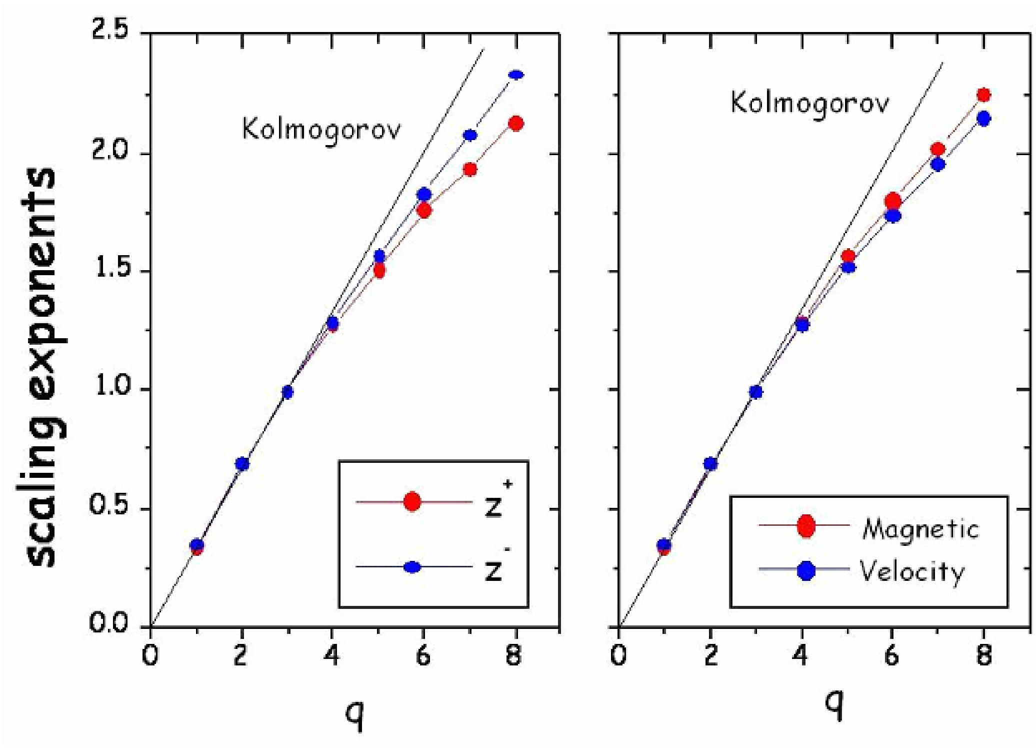
$$u_n(t) = k_n^{-h} F\left[k_n^{1-h}(t - t^*)\right]$$

Time intermittency related to nonlinear dynamics

Scaling of moments of shell fields

$$\left\langle \left[\mathrm{Re}(u_n)\right]^q \right\rangle \approx k_n^{-\zeta_q}$$

Scaling exponents obtained in the range where the flux scales as k_n^{-1}

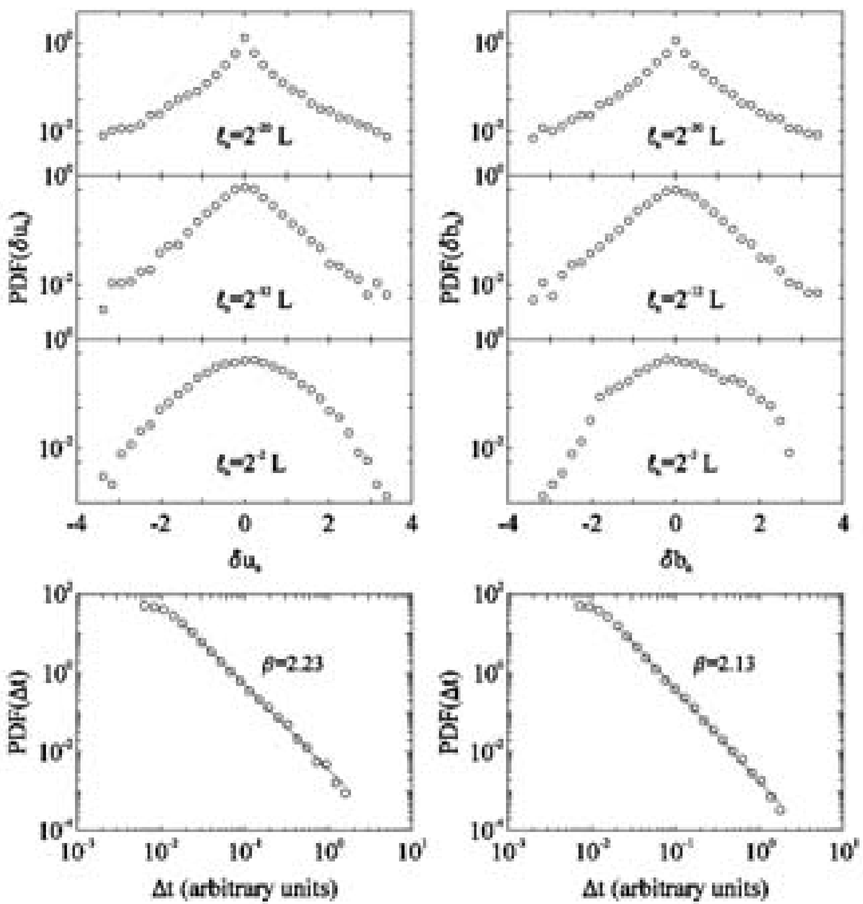


A departure from the Kolmogorov law must be attributed to the "time intermittency" in the shell model.

The departure from the Kolmogorov law measures the "amount" of intermittency

Fields play the same role → the same "amount" of intermittency

Waiting times in the MHD shell model



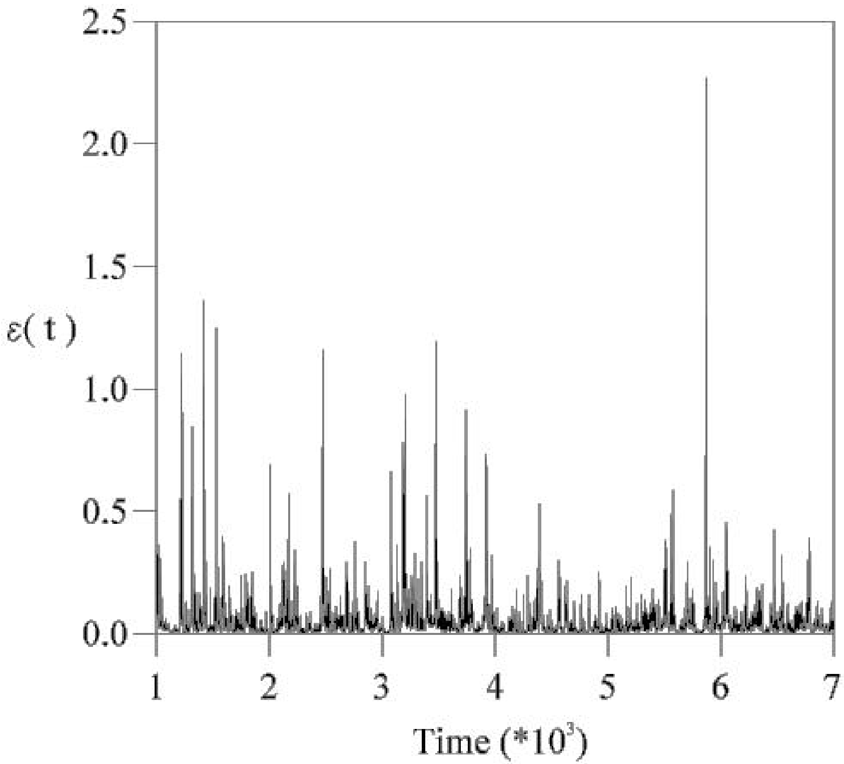
Time intermittency in the shell model is able to capture also that property of real turbulence

Chaotic dynamics generates non poissonian events

Dissipative bursts in shell model

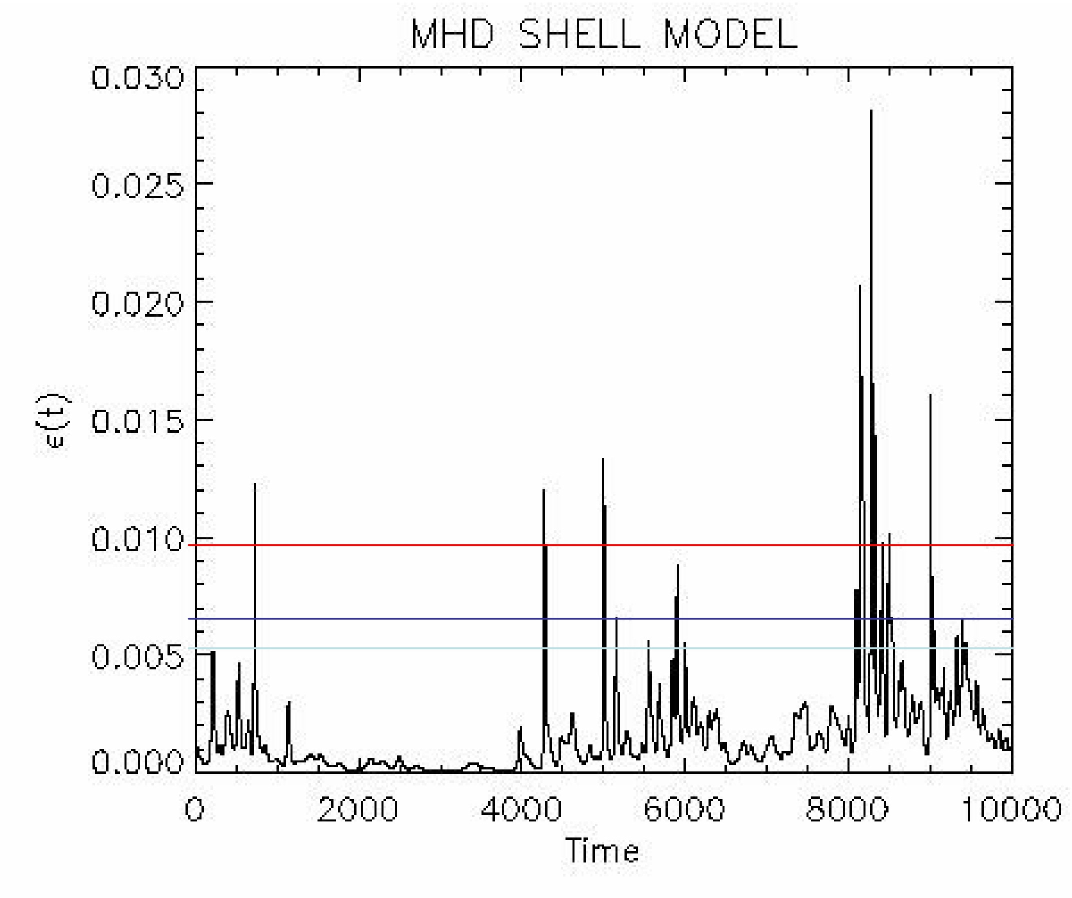
$$\varepsilon(t) = \nu \sum_n k_n^2 |u_n|^2 + \eta \sum_n k_n^2 |b_n|^2$$

The energy dissipation rate is intermittent in time.
Energy is dissipated through impulsive isolated events (bursts).



Inside bursts

Through a threshold process we can identify and isolate each dissipative bursts to make statistics

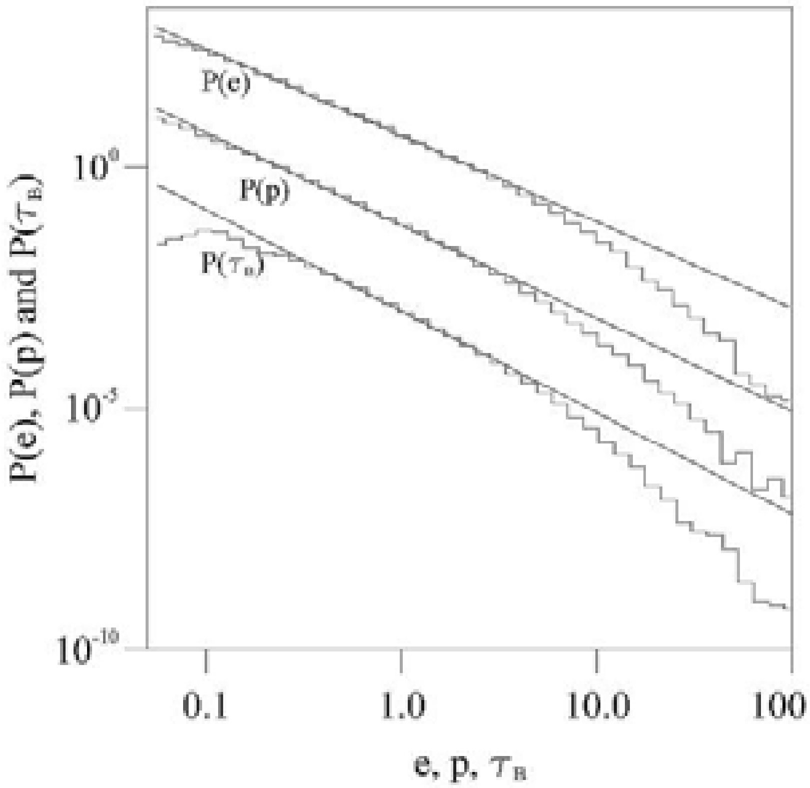


Some statistics

Let us define some statistics on impulsive events

1) Total energy of bursts
2) Time duration
3) Energy of peak

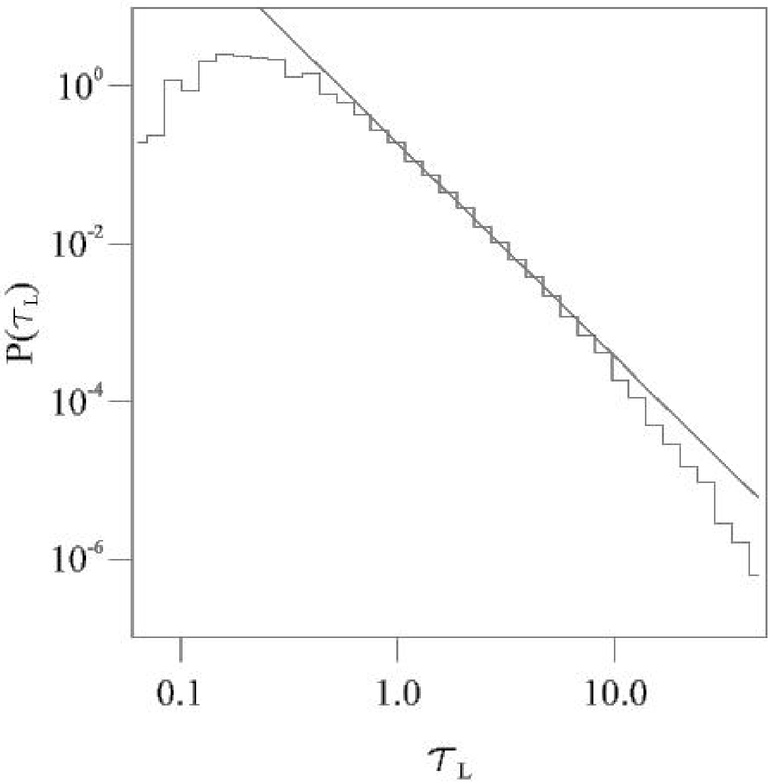
In all cases we found power laws, the scaling exponents depend on threshold.



The waiting times

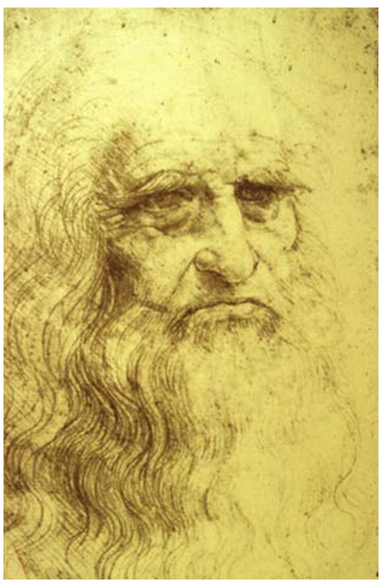
The time between two bursts is t, and let us calculate the pdf p(t).

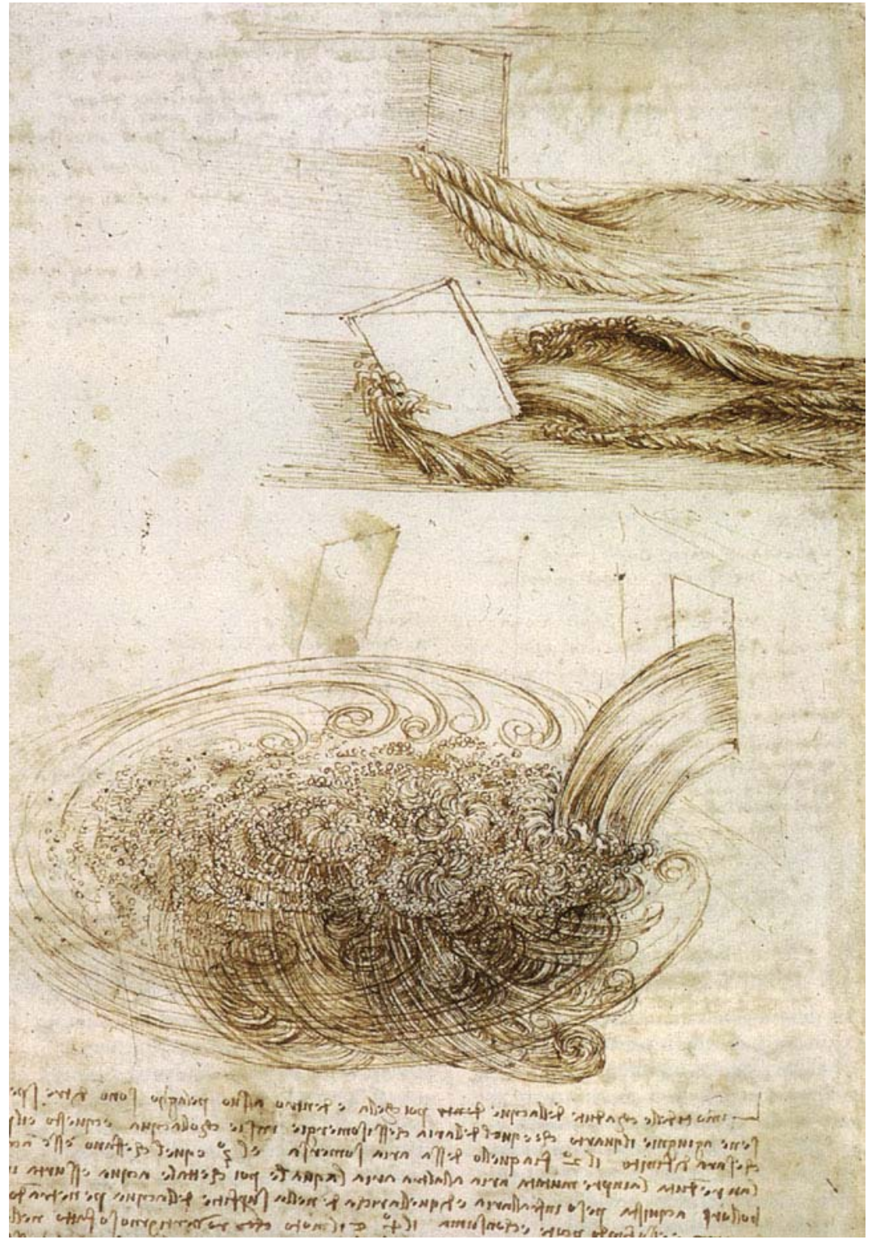
WE FOUND A POWER LAW



Even dissipative bursts are NOT INDEPENDENT

Un'ultima cosa prima di lasciare Leonardo da Vinci





L'osservazione della realtà è solo una questione di "tecnica"?

E' stupefacente come Leonardo riesce a riprodurre la realtà osservata. E' solo bravura "tecnica" o osservava la realtà in modo differente da come possiamo farlo noi?

Ricerca scientifica: stupore e desiderio.

La ricerca scientifica parte dallo stupore per tutto ciò che esiste, e lo stupore diventa desiderio di vedere di più, di scrutare, di osservare nei particolari. E l'osservazione di cose nuove non fa che accrescere lo stupore dell'inizio.

Lasciamo raccontare questa dinamica da Leonardo stesso

"E tirato dalla mia bramosa voglia, vago di vedere la gran copia delle varie e strane forme fatte dalla artifiziosa natura, raggiratomi alquanto infra gli ombrosi scogli, pervenni all'entrata di una gran caverna, dinanzi alla quale restato alquanto stupefatto e ignorante di tal cosa [...] e spesso piegandomi in qua e in là per vedere se dentro vi discernessi alcuna cosa; e questo vietatomi per la grande oscurità che là dentro era. E stato alquanto, subito salse in me due cose, paura e desiderio: paura per la minacciante spelonca, desiderio per vedere se là entro fusse alcuna miracolosa cosa."

(*Leonardo Da Vinci, Scritti letterari*)

La dinamica descritta da Leonardo fa parte della natura non solo dello scienziato, ma di ogni uomo perché suscita una serie di domande. Come riportato in modo mirabile da Leopardi

…e quando miro in cielo arder le stelle;
dico fra me pensando: a che tante facelle?
Che fa l'aria infinita, e quel profondo infinito seren?
Che vuol dir questa solitudine immensa?
ed IO che sono? Così meco ragiono.

Giacomo Leopardi

Canto notturno di un pastore errante dell'Asia