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Computazione quantistica con i fotoni

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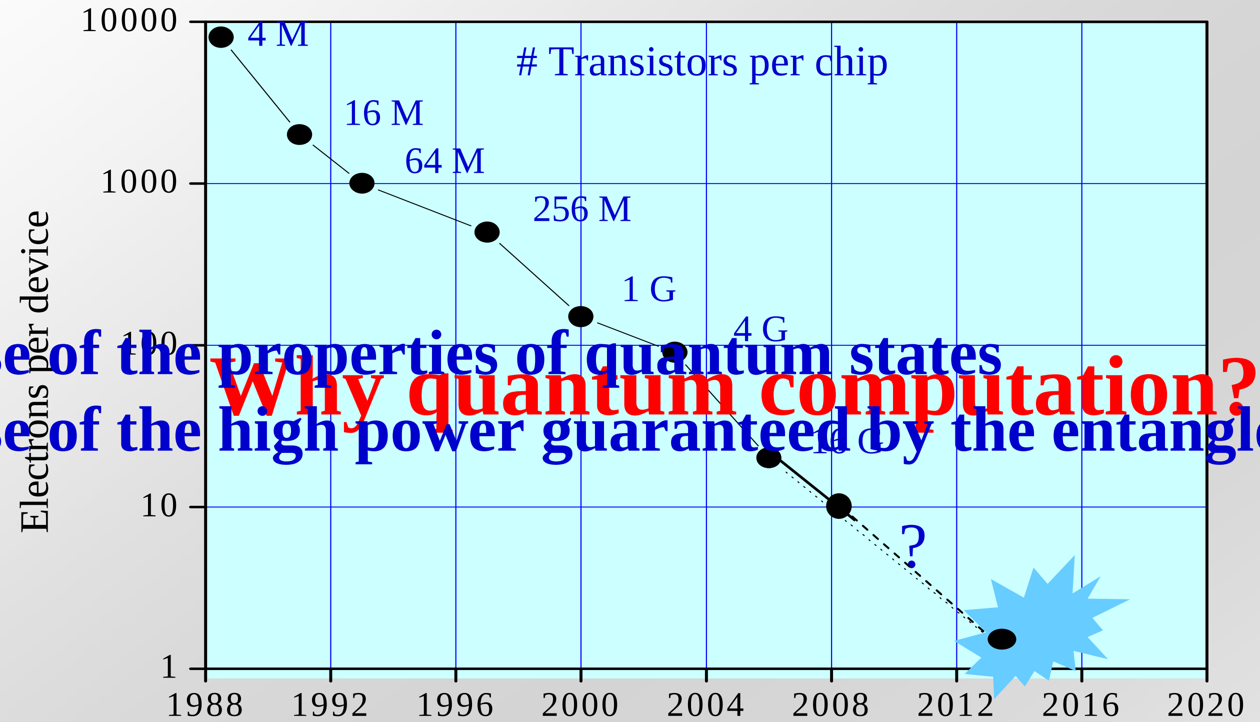
Why quantum computation?

- Because of the properties of quantum states
- Because of the high power guaranteed by the entanglement



By 2015 a single electron can be confined in a transistor

Example: factorizing a 1024-digit number:

- **Classical computer takes a period > universe lifetime**
- **Quantum computer <u>could</u> find the answer in 1sec....**
  (P.W. Shor 1994)

- They are easy to generate, manipulate, transmit and detect
- Have low interaction with the enviroment → low decoherence
- Possible to encode the information in different degrees of freedom of the photons (polarization, momentum, frequency….)

Why quantum computation with photons?

It has been demonstrated that a universal quantum computer can be realized by photons and standard linear optical devices (beam splitters, polarizers, waveplates…..) KLM, Nature 2001

Outline

- **Basic elements**
  quantum bit, quantum register, logic gates, entanglement...

- **Cluster States of Photons**
  properties, One-Way Quantum Computation

- **Spontaneous Parametric Down Conversion**
  the Roma source, tools for measurements with photons

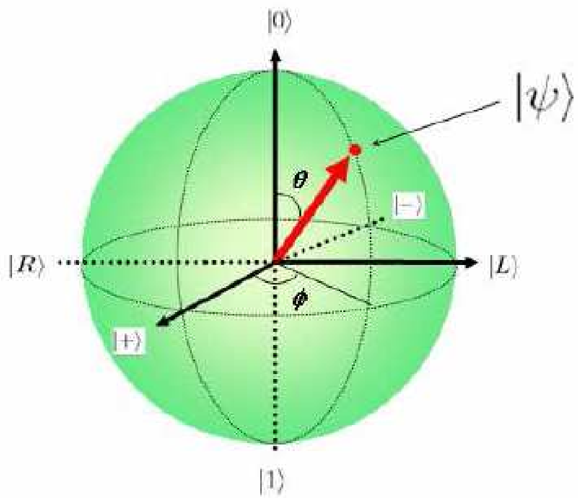
- **One-Way Quantum Computation with photons**
  single qubit rotations, *C-NOT* gate, Grover's search algorithm

- **Optical Quantum Computing in the near future**
  doing now, perspectives

Quantum bit (Qubit)

Coherent **superposition of the orthogonal states |0> and |1>**

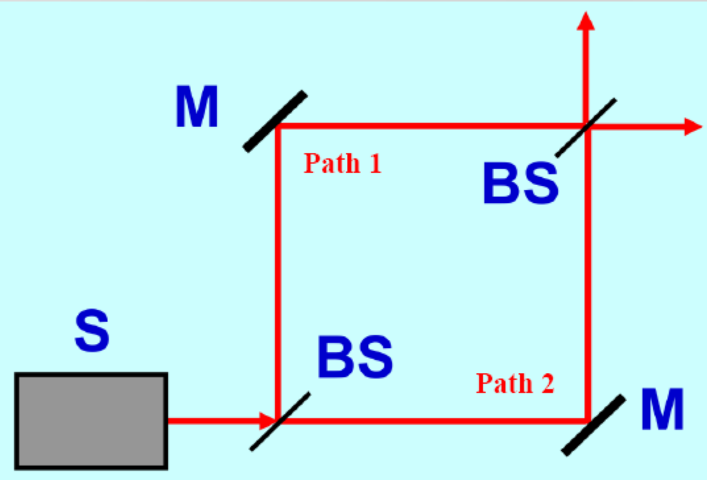
$|Q\rangle = \alpha|0\rangle + \beta|1\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$)



$$|+(-)\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$|L(R)\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$$

$$|\psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle\right),$$



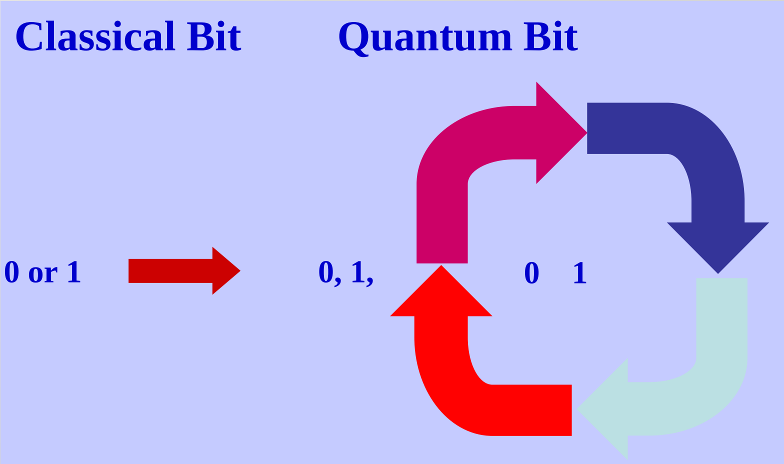
Example:

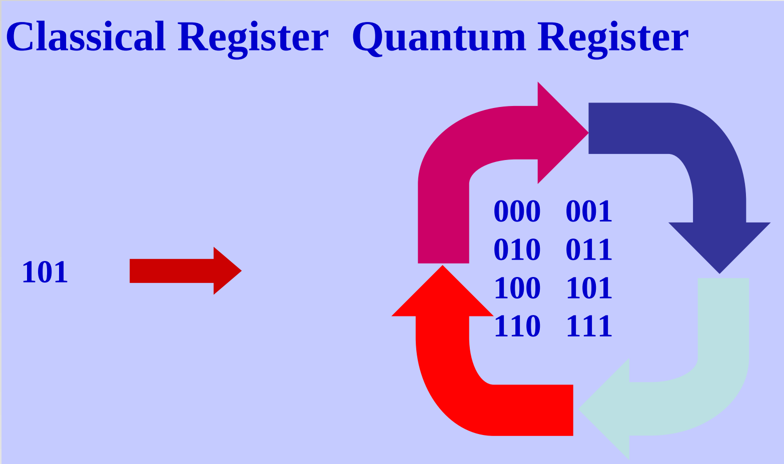
- **photon passing through a Mach-Zehnder interferometer: $|Q\rangle = \alpha|\text{Path 1}\rangle + \beta|\text{Path 2}\rangle$**
- **superposition of H and V polarization: $|Q\rangle = \alpha|H\rangle + \beta|V\rangle$**

Quantum register (3-bit register)

Classical: can store exactly one of the eight different numbers, 000, 001, 010, ….., 111

Quantum: can store up to 8 numbers in a quantum superposition $\rightarrow$ N qubits: up to 2^N numbers at once





Logic gates (1)

Single qubit gate: linear operator in a 2-dimension space
Complex 2x2 unitary matrix

$$U = e^{-i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}} \qquad \vec{\sigma} = (X, Y, Z)$$

$$|\psi_{in}\rangle \xrightarrow{\;U\;} |\psi_{out}\rangle$$

***NOT*: *X* = σ_x** **Y = σ_Y** **Z = σ_Z**

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = i\begin{pmatrix} -\beta \\ \alpha \end{pmatrix} = \begin{pmatrix} -\beta \\ \alpha \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$$

Any kind of qubit rotation in the Bloch sphere can be realized by combining in different ways the three Pauli matrices

Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

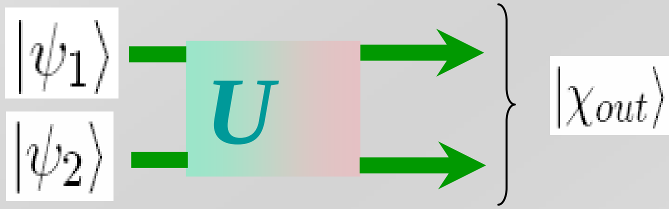
$$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} \equiv |+\rangle$$

$$|1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}} \equiv |-\rangle$$

Logic gates (2)

Two qubit gates: unitary 4x4 matrices

$$C\text{-}U = |0\rangle_c\langle 0| \otimes \mathbb{1}_t + |1\rangle_c\langle 1| \otimes U_t$$



Quantum vs. classic

- **<u>Classical case:</u> *any* kind of logic gate can be realized by suitable combinations of the NAND gate.**
- **<u>Quantum case:</u> *any* N-qubit logic gate can be realized by 1-qubit gates and one 2-qubit gate, (C-PHASE, C-NOT)**

C-PHASE:

$$CP = |0\rangle_c\langle 0| \otimes 1_t + |1\rangle_c\langle 1| \otimes (\sigma_Z)_t$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Logic gates (3)

$$C - NOT \Rightarrow U_t \equiv (\sigma_X)_t$$

$$C_{NOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Control	Target	Control	Target
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

C-NOT can generate entanglement:

$$C_{NOT}\left(|+\rangle|1\rangle\right) = C_{NOT}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)|1\rangle =$$

$$C_{NOT}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} = \frac{|0.0\rangle + |1,1\rangle}{\sqrt{2}}$$

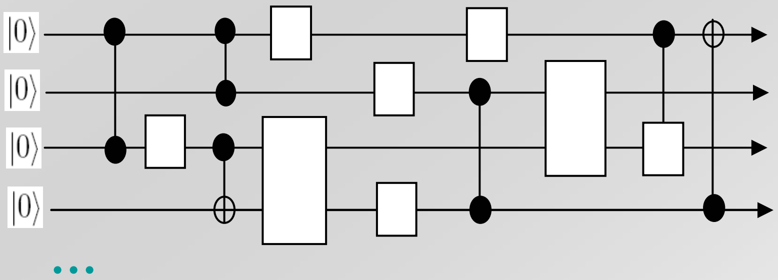
Circuital model of a quantum computer

- **Superposition:**

$$\left|\Psi\right\rangle = \sum_{i_{n-1}=0}^{1} \dots \sum_{i_1=0}^{1} \sum_{i_0=0}^{1} c_{n-1\dots i_1, i_o} \left|i_{n-1}\right\rangle \otimes \dots \otimes \left|i_0\right\rangle$$

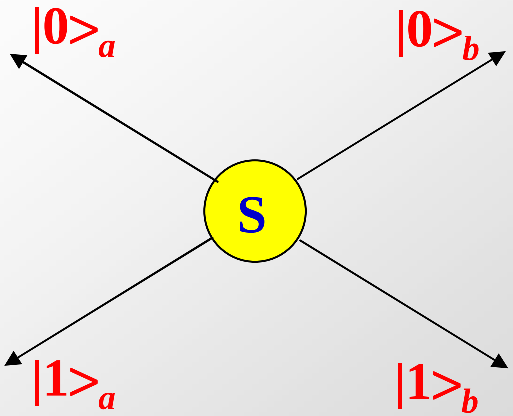
- **Parallelism**

- **Unitary evolution of |Ψ> based on single and two qubit logic gates**



Linear Optics Quantum Computation:
based on single photon qubits, linear optics devices for single qubit rotations and two qubit gates (KLM, Nature ‘01)

Entanglement



Left: particle "a" carries the information "0", or vice versa.

Right: particle "b" carries the information "1", or vice versa.

$$\left|\Psi\right\rangle_{ab} = \frac{\left|0\right\rangle_a \left|1\right\rangle_b \pm \left|1\right\rangle_a \left|0\right\rangle_b}{\sqrt{2}}$$

can not be expressed by the product of single qubit states $|\Psi\rangle_a$ and $|\Psi\rangle_b$

Neither of the two qubits carries a definite value:
as soon as one qubit is measured randomly, the other one will immediately be found to carry the opposite value, *independently of the relative distance* (quantum nonlocality)

Quantum nonlocality

Singlet state:

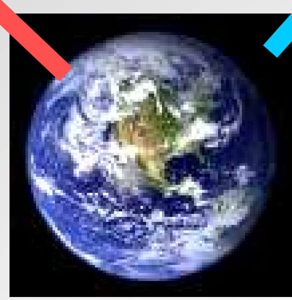
$$\left|\Psi^{-}\right\rangle_{ab} \frac{1}{\sqrt{2}}\left(\left|H\right\rangle_{a}\left|V\right\rangle_{b} - \left|V\right\rangle_{a}\left|H\right\rangle_{b}\right)$$

ALICE

a

BOB

b



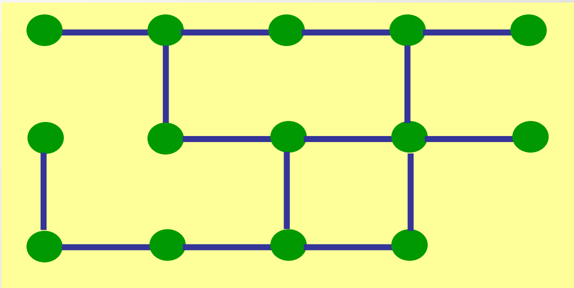
Alice measures photon *a* with 50% probability to detect:

- **H or V (|0> or |1>):** ↔ , ↕
- **45° or -45° (|+> or |->):** ⤢, ⤡
- ***L* or *R*:** ⟲, ⟳

Perfect correlations in any basis!

Cluster states in Quantum Information

Particular graph states associated to a n-dimensional lattice

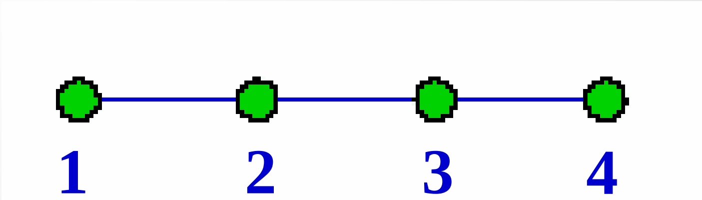


Each dots correspond to the qubit:

Each link corresponds to a Control σ_Z gate

- **Create a genuine multiqubit entanglement**
- **Robust entanglement against single qubit measurements**
- **Fundamental resource for one-way quantum computation**

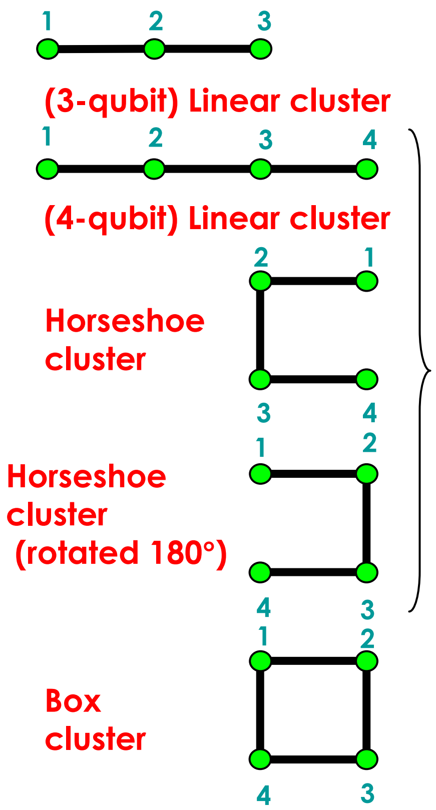
4-qubit linear cluster states



$$|C_4\rangle = \frac{1}{2}\left(|+00+\rangle + |+01-\rangle + |-10+\rangle - |-11-\rangle\right)$$

$$\boxed{|C_4\rangle \neq \frac{1}{\sqrt{2}}\left(|+0\rangle + |-1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0+\rangle - |1-\rangle\right)}$$

Not factorizable!



$$|+\rangle_1|0\rangle_2|+\rangle_3$$
$$+\ |-\rangle_1|1\rangle_2|-$$

$$|+\rangle_1|0\rangle_2|0\rangle_3|+\rangle_4$$
$$+\ |+\rangle_1|0\rangle_2|1\rangle_3|-\rangle_4$$
$$+\ |-\rangle_1|1\rangle_2|0\rangle_3|+\rangle_4$$
$$-\ |-\rangle_1|1\rangle_2|1\rangle_3|-\rangle_4$$

$$|+\rangle_1|0\rangle_2|0\rangle_3|+\rangle_4$$
$$+\ |-\rangle_1|0\rangle_2|1\rangle_3|-\rangle_4$$
$$+\ |-\rangle_1|1\rangle_2|0\rangle_3|-\rangle_4$$
$$+\ |+\rangle_1|1\rangle_2|1\rangle_3|+\rangle_4$$

One-way quantum computation

(Briegel *et al.* PRL 01)

Initialization

- Preparation of the cluster state

Manipulation

- Algorithm: pattern of single qubit measurements
  Qubit j measured in the bases:

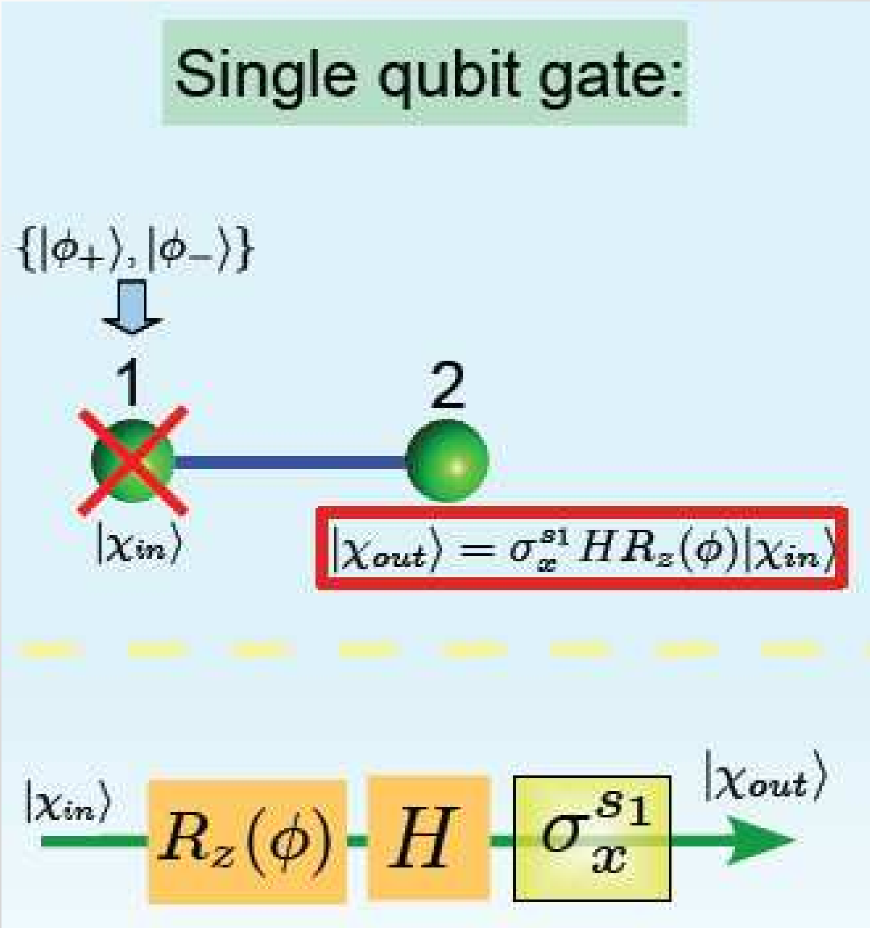
$$|\phi_\pm\rangle_j = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{-i\phi}|1\rangle)$$

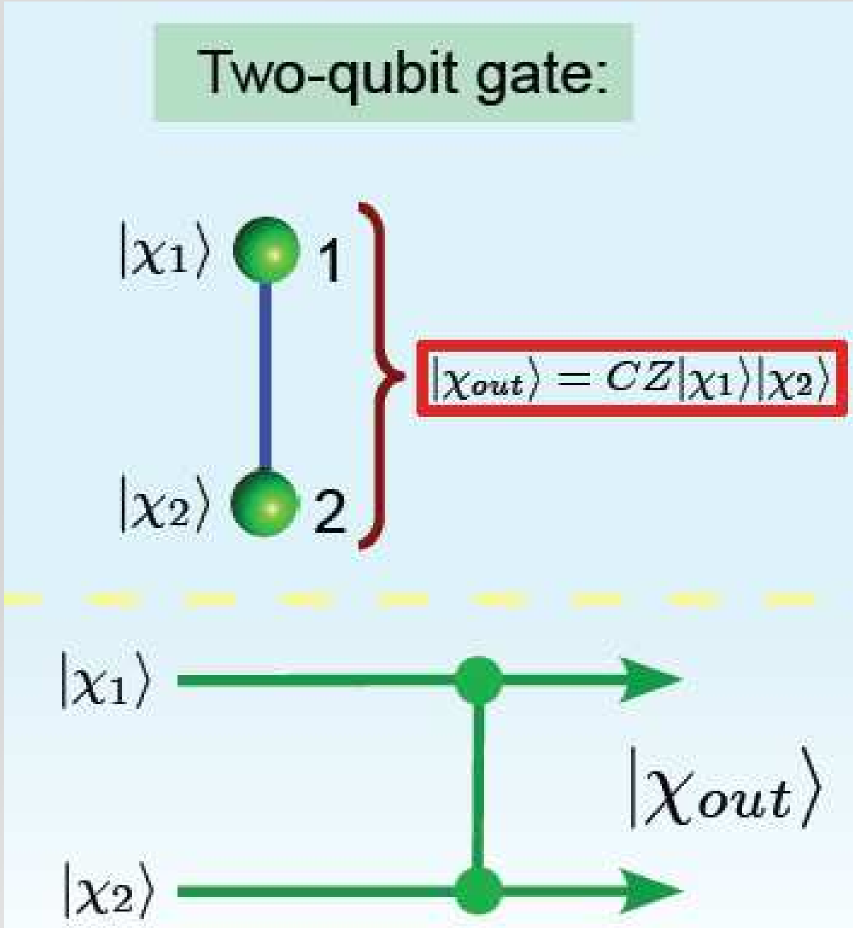
- Feed forward measurements
- Irreversibility (one-way)

Read out

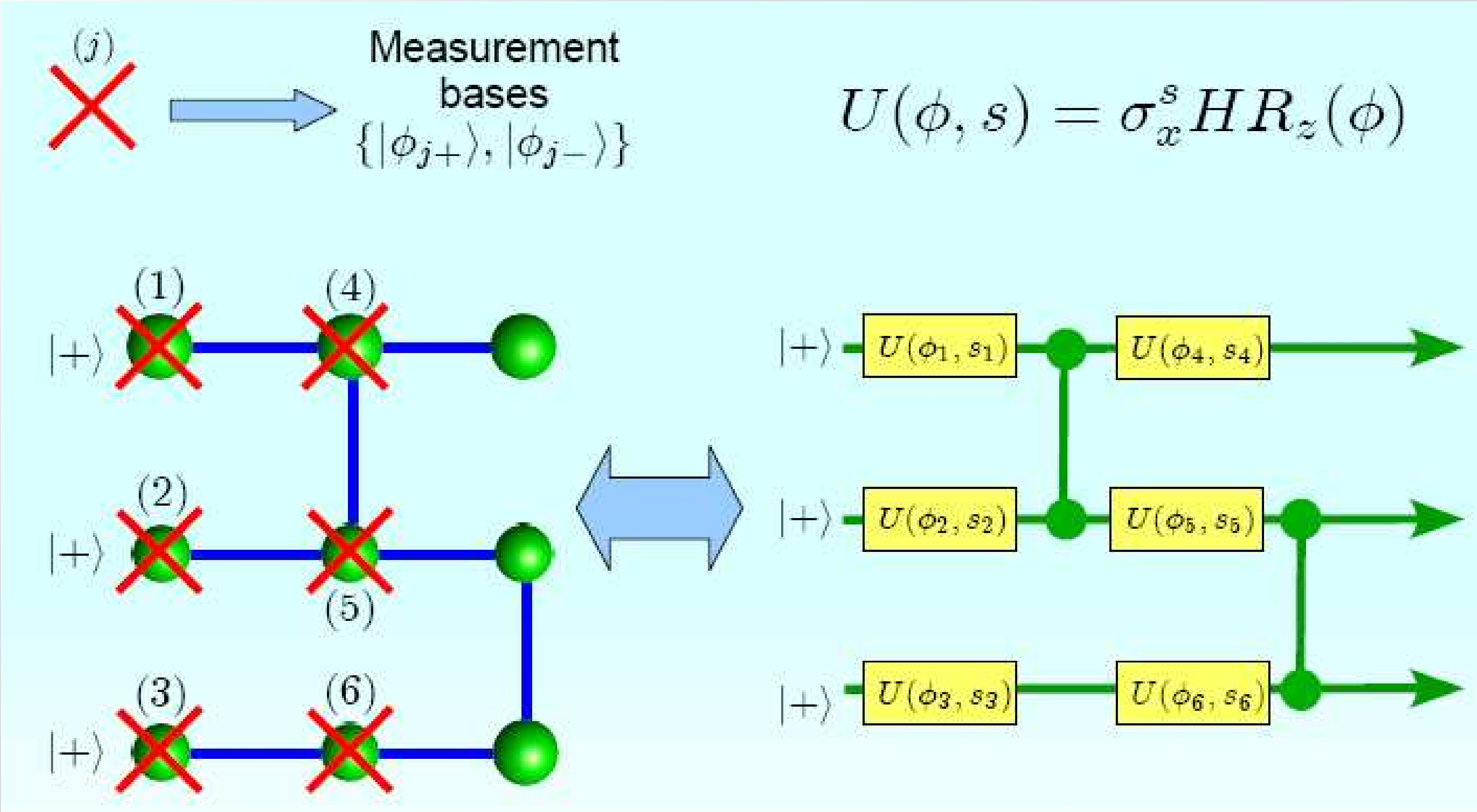
- Feed forward corrections
- Not measured qubit: output

Building blocks of the logical operations



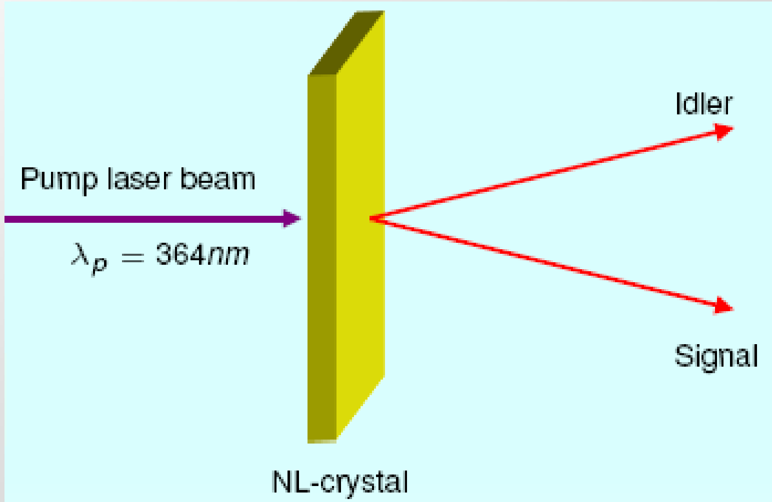


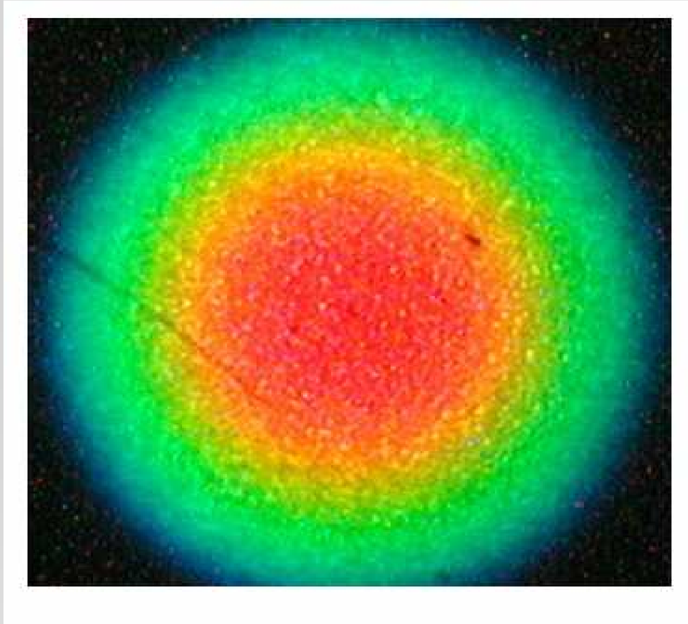
Logical operation: example



Entangled states with photons

Allows to generate photon pairs by the spontaneous parametric down conversion (SPDC) process



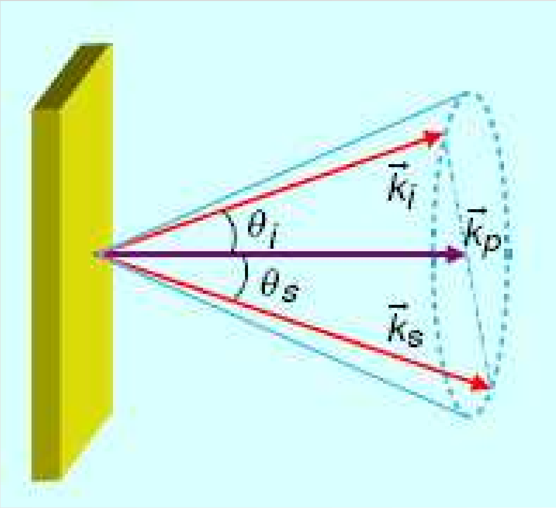


Twin photons created over conical regions, at different wavelengths, with polarization orthogonal to that of the pump

SPDC features

- Low probability ($\cong 10^{-9}$)
- Non-deterministic process
- Energy matching:

$$\hbar\omega_p = \hbar\omega_i + \hbar\omega_s \Rightarrow \quad \frac{1}{\lambda_p} = \frac{1}{\lambda_i} + \frac{1}{\lambda_s}$$

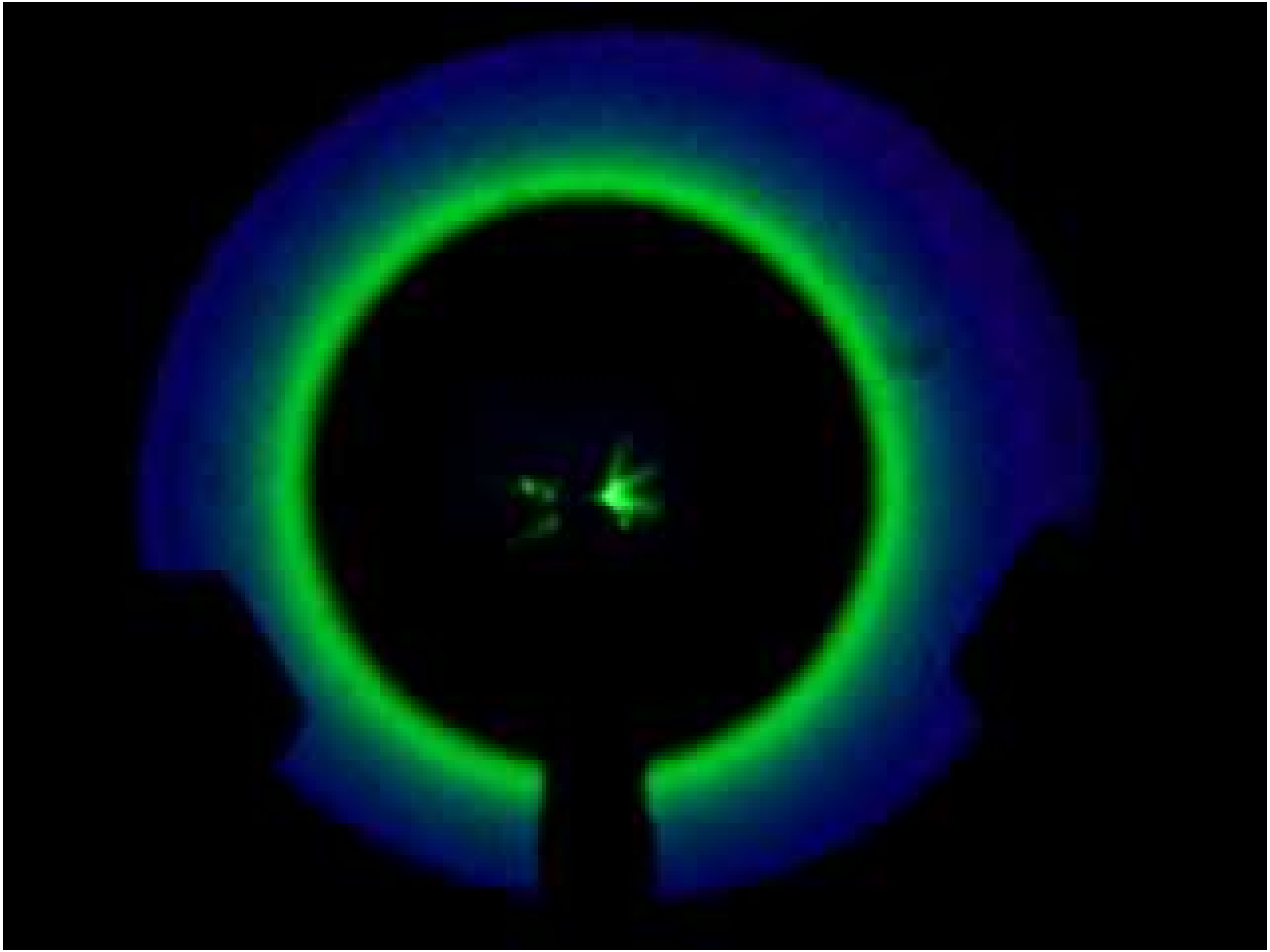


- Phase matching:

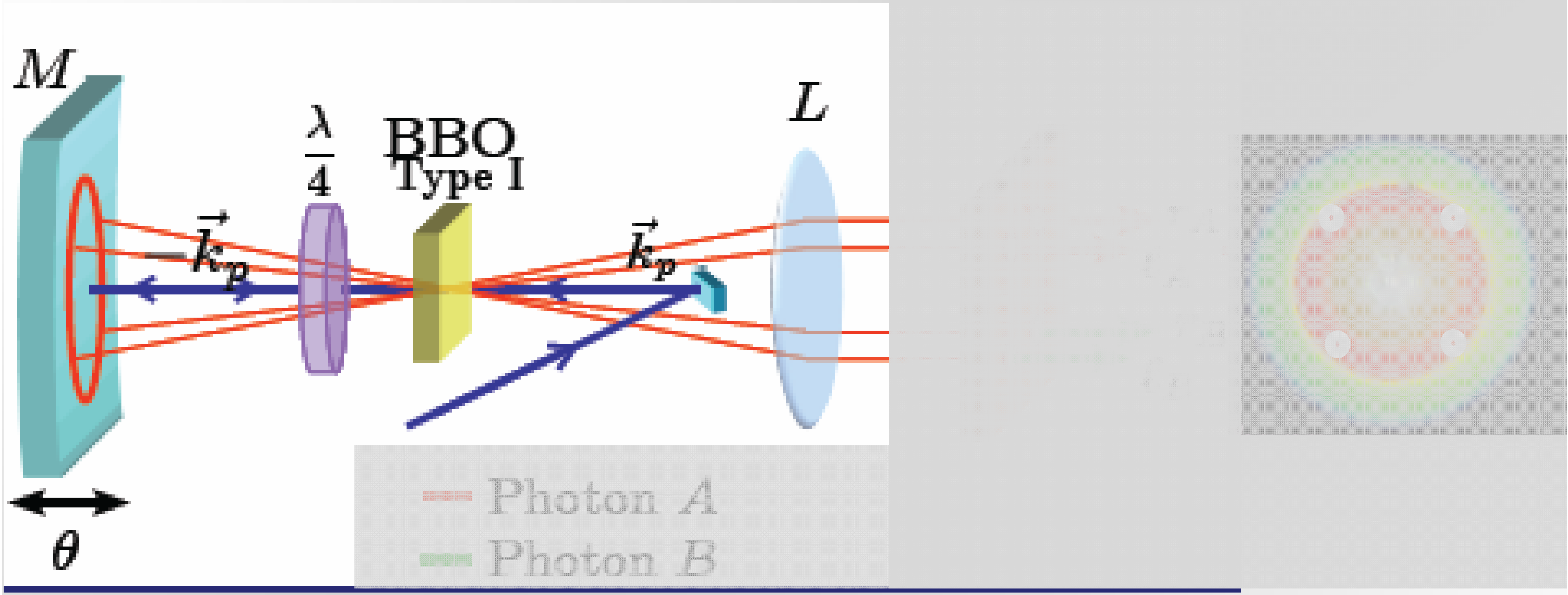
$$\vec{k}_p = \vec{k}_i + \vec{k}_s$$

- Degenerate emission:

$$\lambda_i = \lambda_s = 2\lambda_p, \quad \theta_i = \theta_s, \quad |\vec{k}_i| = |\vec{k}_s| \Rightarrow \text{emission cone}$$



The Roma source: polarization – momentum hyperentanglement of 2 photons

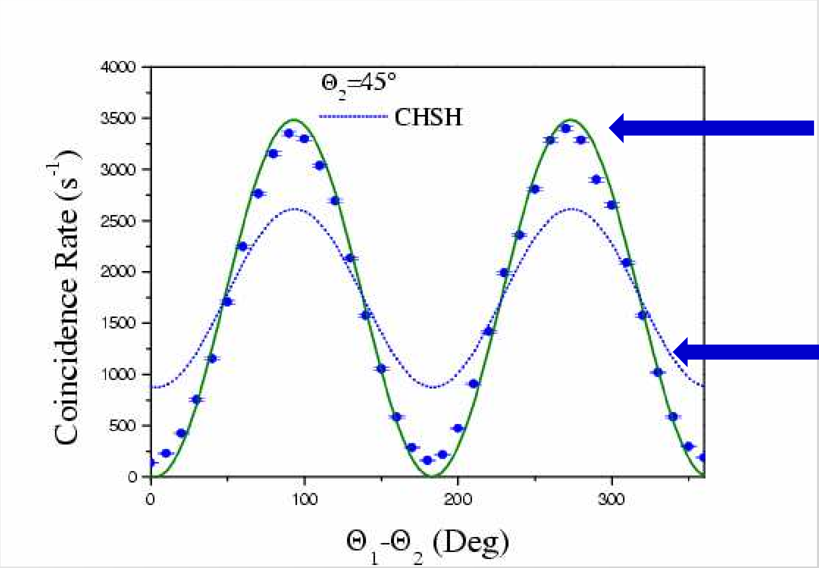


$$|\Pi\rangle = \frac{1}{\sqrt{2}}\left[|H_a H_b\rangle + e^{i\theta}|V_a V_b\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|l_a r_b\rangle + e^{i\phi}|r_a l_b\rangle\right] = |\Phi\rangle \otimes |\psi\rangle$$

Barbieri *et al.* PRA 05
Cinelli *et al.* PRL 05
Barbieri *et al.* PRL 06

2 photons → 4 qubits

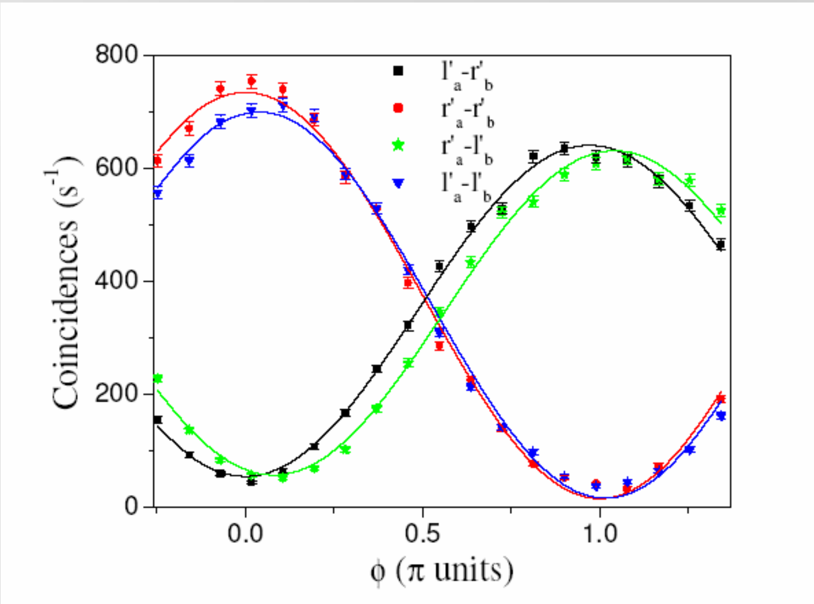
Polarization – momentum entanglement



Quantum

Classical

Bell-CHSH inequality test: 213-σ violation



Bell-CHSH inequality test: 170-σ violation

Photon cluster states

4-photon cluster states (based on the simultaneous generation of 2 photon pairs **[Zeilinger *et al.*, Nature (05, 07)]**

$$\frac{1}{4}\left[\left|H_aH_bH_cH_d\right\rangle+\left|H_aH_bV_cV_d\right\rangle+\left|V_aV_bH_cH_d\right\rangle-\left|V_aV_bV_cV_d\right\rangle\right]$$

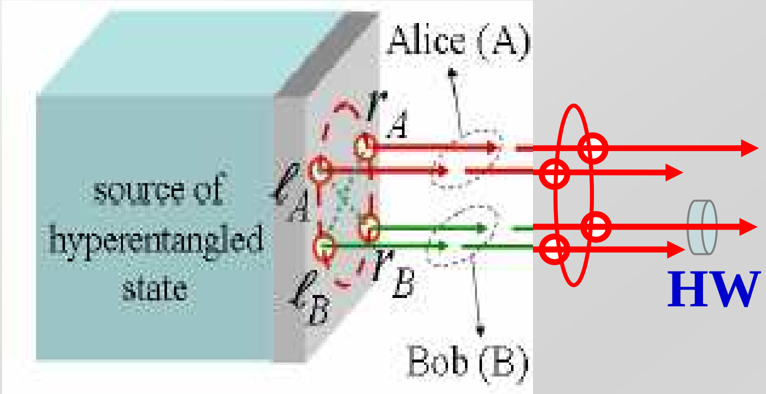
Problems:

- Generation/detection rate ~ 1 Hz
- Limited purity of the state
- Need of post-selection

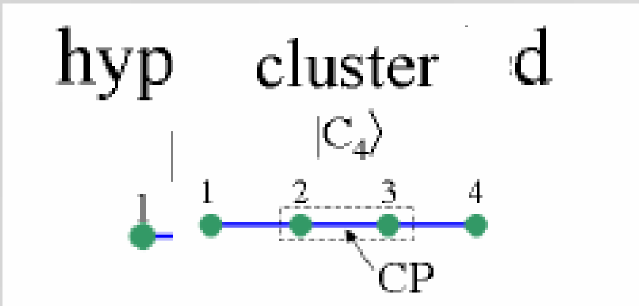
Alternative:

Generate cluster states starting from 2-photon hyperentangled states

From hyperentangled to cluster states



$$\frac{1}{4}\left[\left|\frac{1}{4}\left[\left|H_a l_a\right\rangle\left|H_b r_b\right\rangle+\left|H_a r_a\right\rangle\left|H_b l_b\right\rangle+\left|V_a l_a\right\rangle\left|V_b r_b\right\rangle-\left|V_a r_a\right\rangle\left|V_b l_b\right\rangle\right], l_b\right\rangle\right]$$

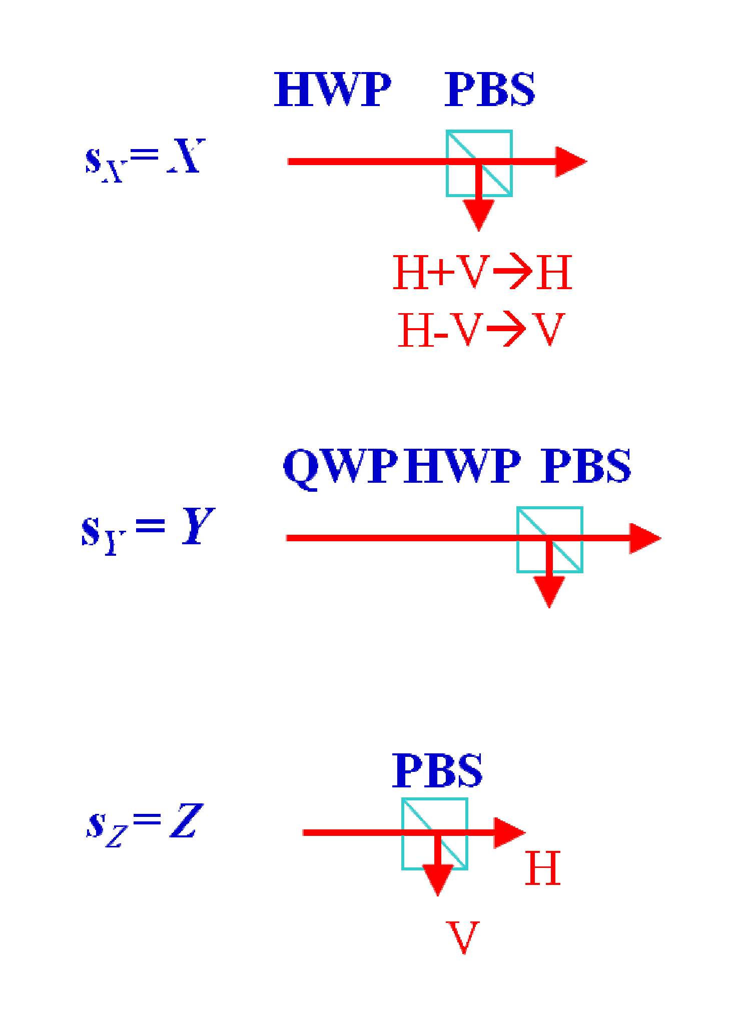


- High generation rate (~1000 coincidences per sec detected)
- High purity of the states
- No post-selection required

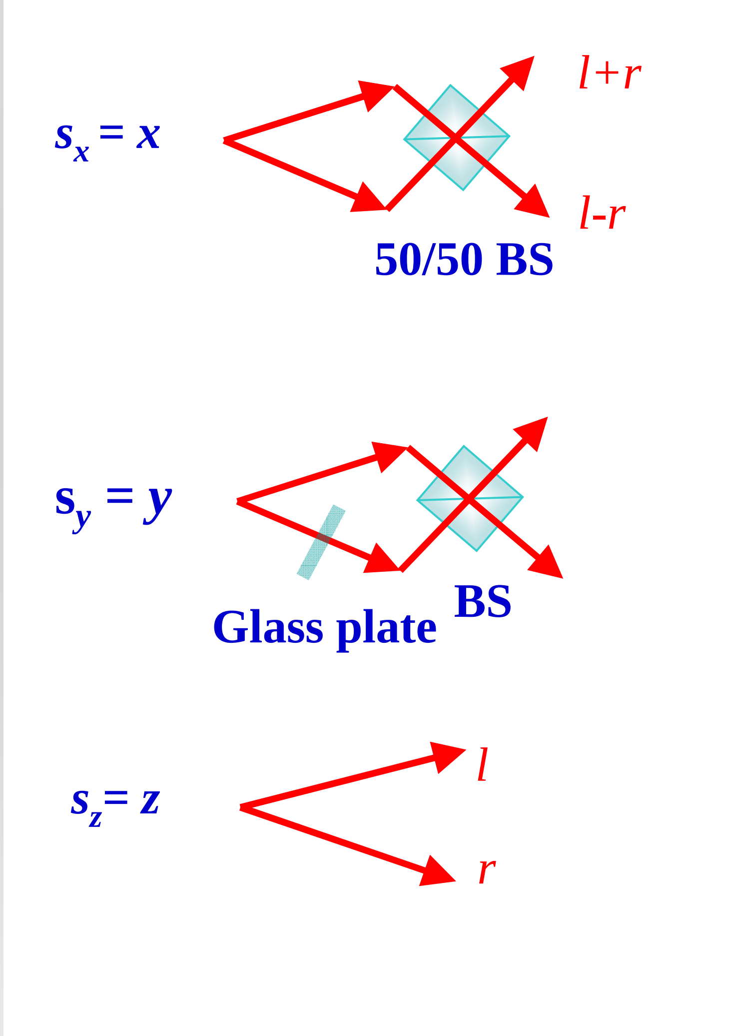
Vallone *et al.* PRL 07

Measurement tools

Polarization (p) observables

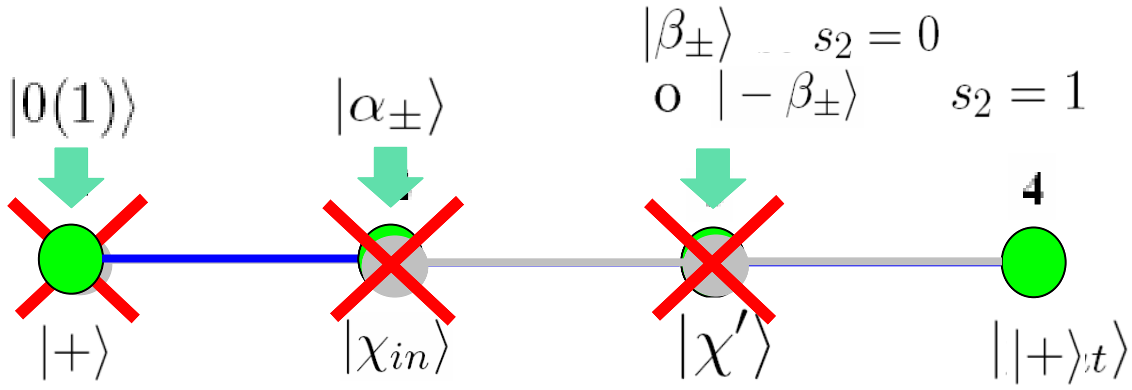


Momentum (k) observables



Single qubit rotation

$$H = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$$
$$R_z(\alpha) = e^{-\frac{i}{2}\alpha\sigma_z}$$
$$R_x(\beta) = e^{-\frac{i}{2}\beta\sigma_x}$$

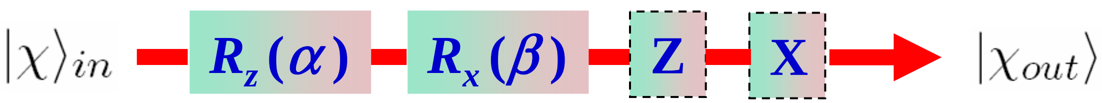


By choosing α and β any arbitrary single qubit rotation can be performed up to Pauli errors (corrected by feed-forward)

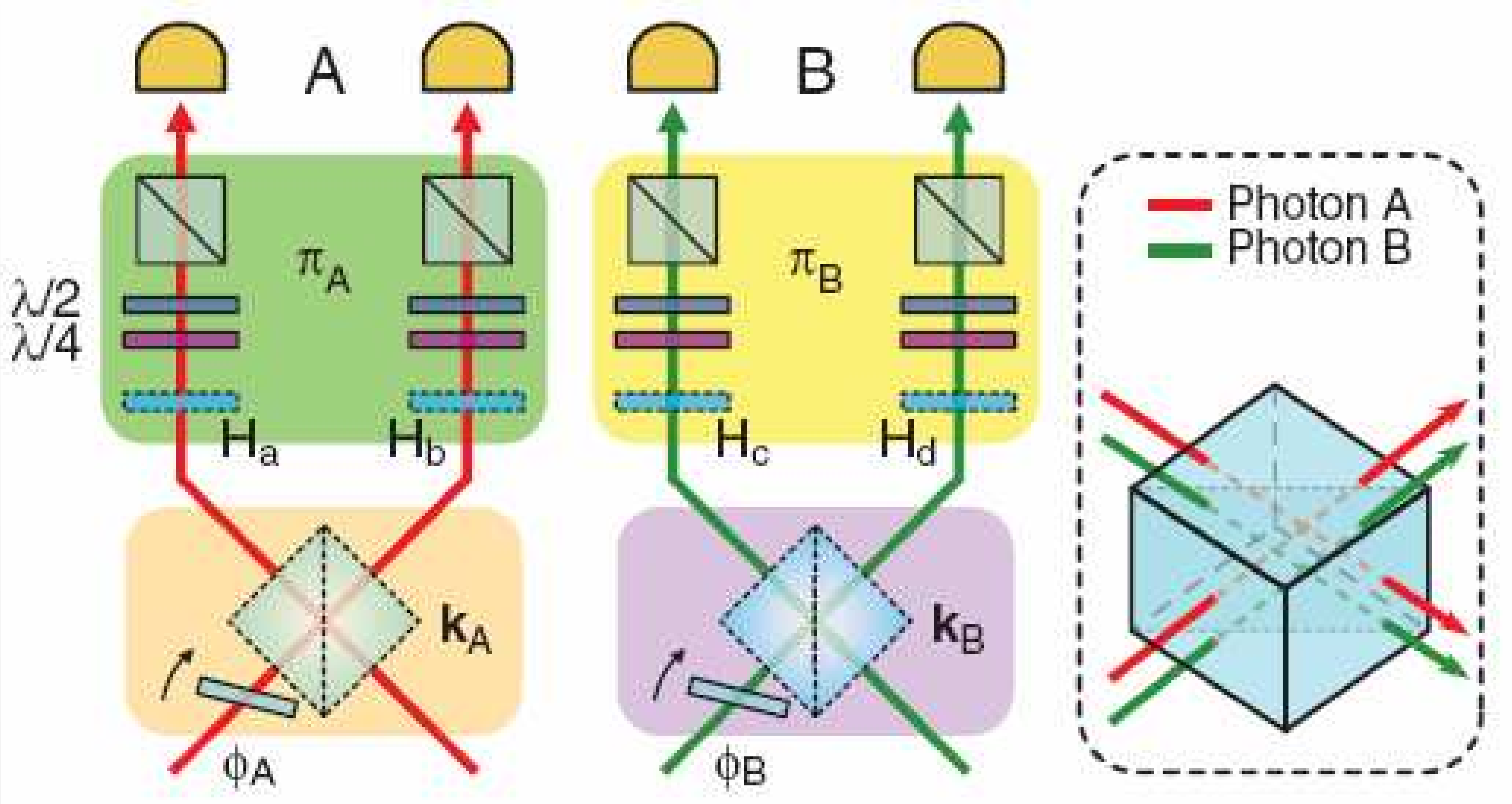
I) $|\chi_{in}\rangle = Z^{s_1}|+\rangle$

II) $|\chi'\rangle = X^{s_2} H R_z(\alpha)|\chi_{in}\rangle$

III) $$|\chi_{out}\rangle = X^{s_3} H R_z\left[(-1)^{s_2}\beta\right]|\chi'\rangle = X^{s_3} Z^{s_2} R_x(\beta) R_z(\alpha)|\chi_{in}\rangle$$



Measurement setup: probabilistic QC



- Measurements done by spatial mode matching on a common 50:50 BS
- Qubit rotations performed by using either π or k as output qubit

Vallone *et al.* LPL 08

Polarization output qubit

output state:

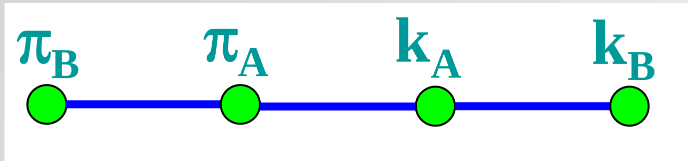
$$|\chi_{out}\rangle_{\pi_B} = Z^{s_3} X^{s_2} H R_x(\beta) R_z(\alpha) |\chi_{in}\rangle$$

$s_2 = s_3 = 0$:

$$|\chi^{00}_{out}\rangle_{\pi_B} = H R_x(\beta) R_z(\alpha) |+\rangle$$

$$\begin{cases} |\chi^{01}_{out}\rangle_{\pi_B} = Z |\chi^{00}_{out}\rangle_{\pi_B} \\ |\chi^{10}_{out}\rangle_{\pi_B} = X |\chi^{00}_{out}\rangle_{\pi_B} \\ |\chi^{11}_{out}\rangle_{\pi_B} = ZX |\chi^{00}_{out}\rangle_{\pi_B} \end{cases}$$

Linear momentum output qubit



output state:

$$|\chi_{out}\rangle_{\mathbf{k_B}} = Z^{s_3+1} X^{s_2} H R_x[(-1)^{s_2}\beta] R_z(\alpha) |\chi_{in}\rangle$$

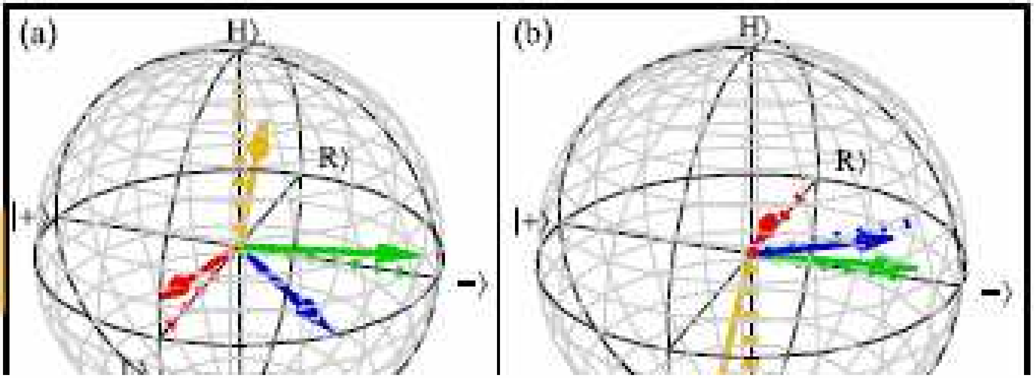
$s_2 = s_3 = 0$:

$$|\chi^{00}_{out}\rangle_{\mathbf{k_B}} = Z H R_z(\alpha) |+\rangle$$

$$\begin{cases} |\chi^{01}_{out}\rangle_{\mathbf{k_B}} = Z |\chi^{00}_{out}\rangle_{\mathbf{k_B}} \\ |\chi^{10}_{out}\rangle_{\mathbf{k_B}} = X |\chi^{00}_{out}\rangle_{\mathbf{k_B}} \\ |\chi^{11}_{out}\rangle_{\mathbf{k_B}} = XZ |\chi^{00}_{out}\rangle_{\mathbf{k_B}} \end{cases}$$

Experimental results with probabilistic QC

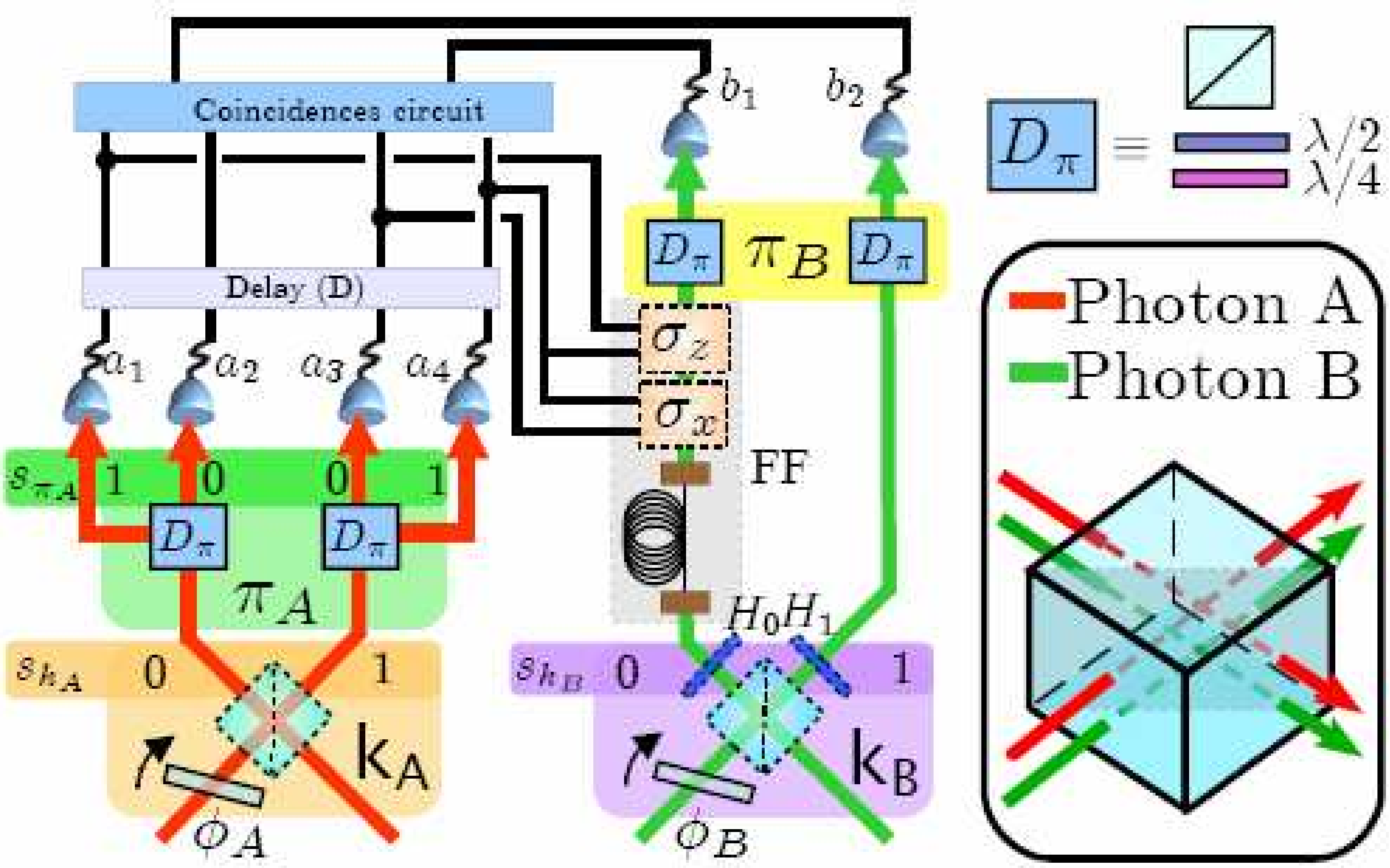
π_B



	α	β	F $(s_2 = s_3 = 0)$	F $(s_2 = 1, s_3 = 0)$	Colour
π_B	0	$\pi/2$	0.908 ± 0.006	0.928 ± 0.013	orange
	$-\pi/2$	0	0.942 ± 0.004	0.902 ± 0.007	red
	$-\pi/2$	$\pi/2$	0.913 ± 0.005	0.904 ± 0.010	green
	$-\pi/2$	$-\pi/4$	0.942 ± 0.006	0.955 ± 0.012	blue

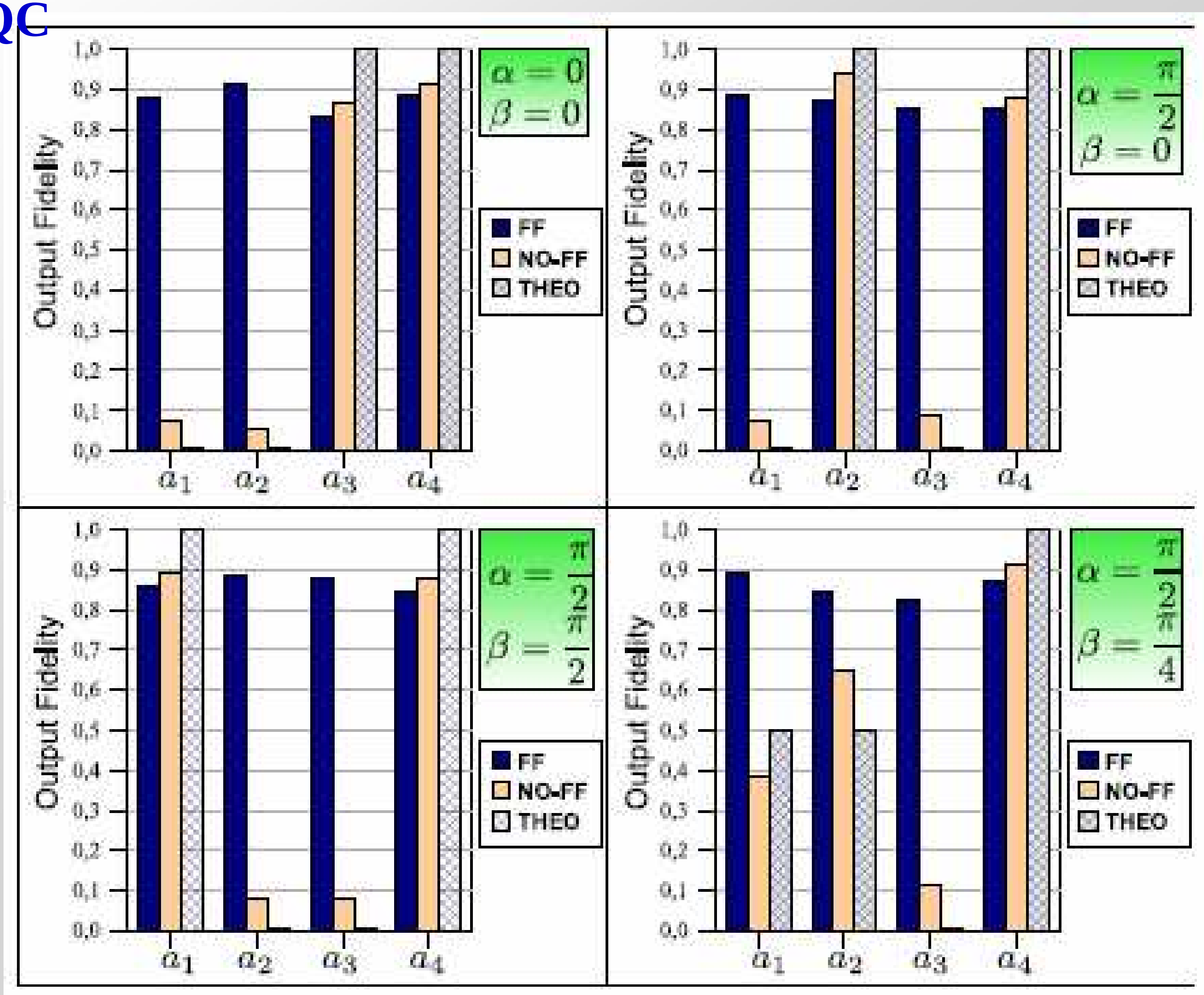
	$\alpha(\beta = 0)$	F $(s_2 = s_3 = 0)$	F $(s_2 = 0, s_3 = 1)$	Colour
k_B	0	0.961 ± 0.003	0.971 ± 0.003	red
	$\pi/2$	0.879 ± 0.006	0.895 ± 0.005	green
	$\pi/4$	0.998 ± 0.005	0.961 ± 0.006	orange
	$-\pi/4$	0.833 ± 0.007	0.956 ± 0.006	blue

Measurement setup: deterministic QC

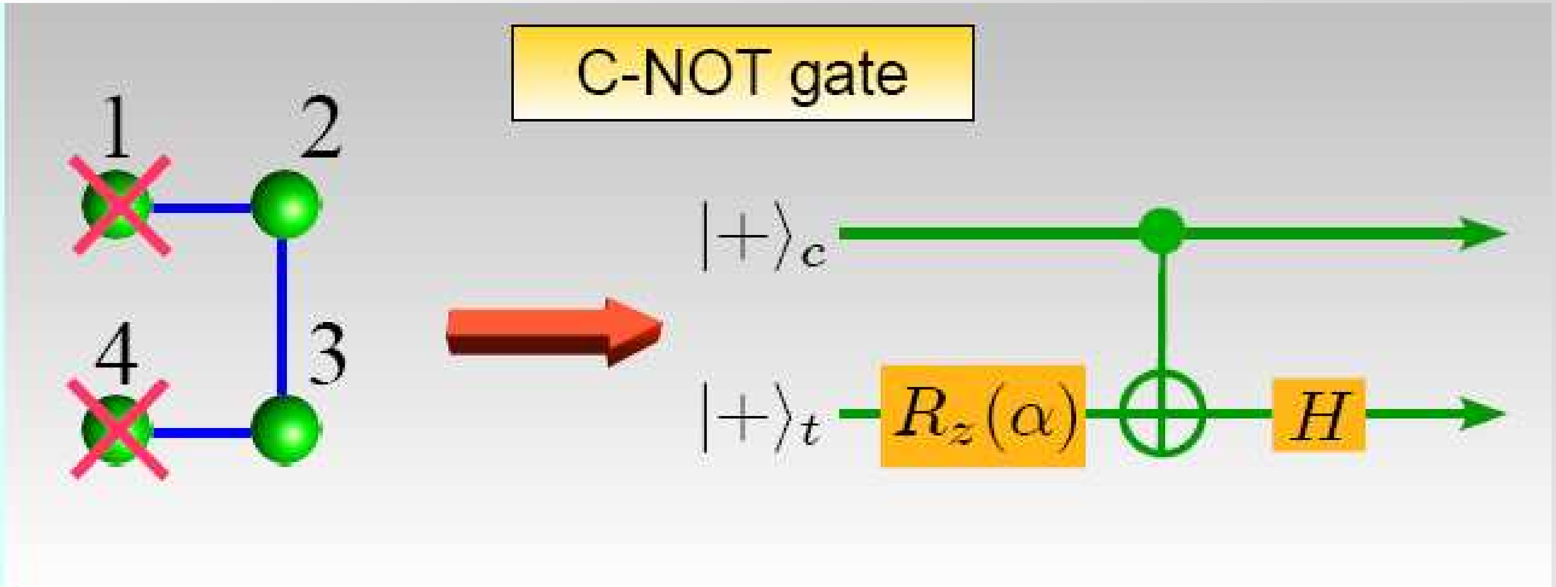


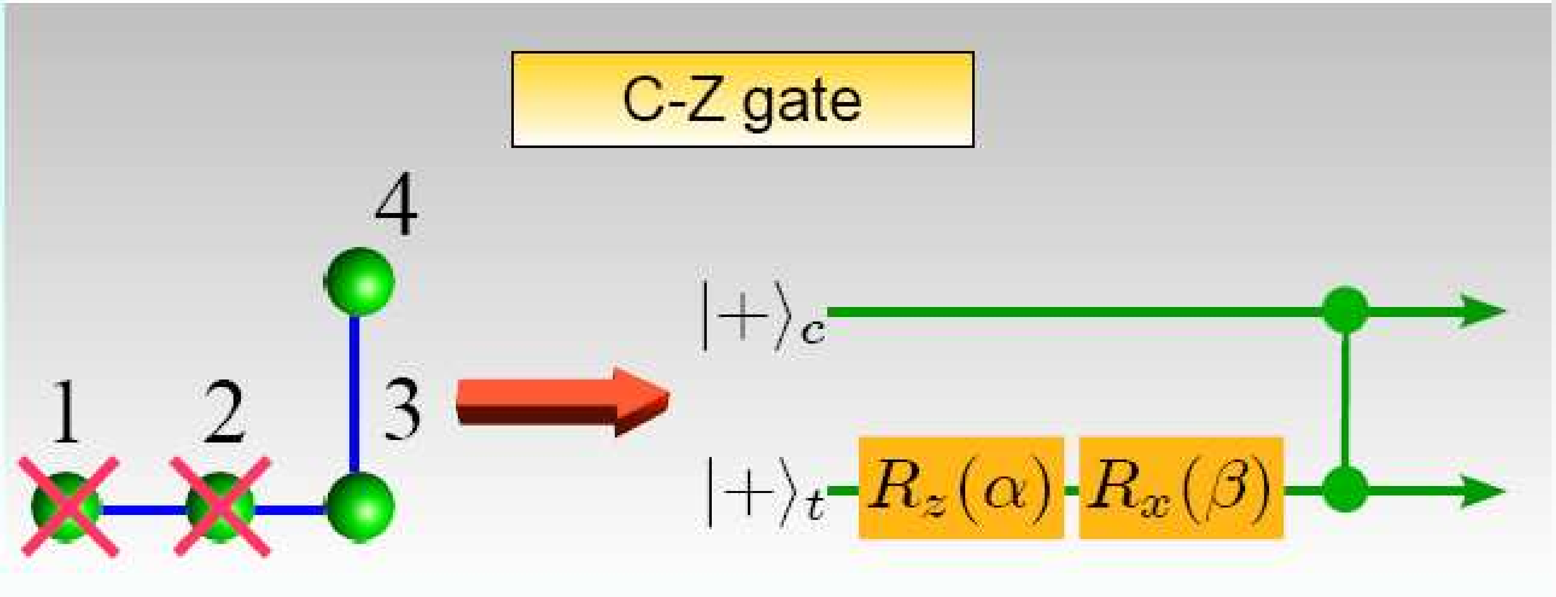
Vallone *et al.* PRL 08

Experimental results with deterministic QC



2-qubit gates





C-NOT gate

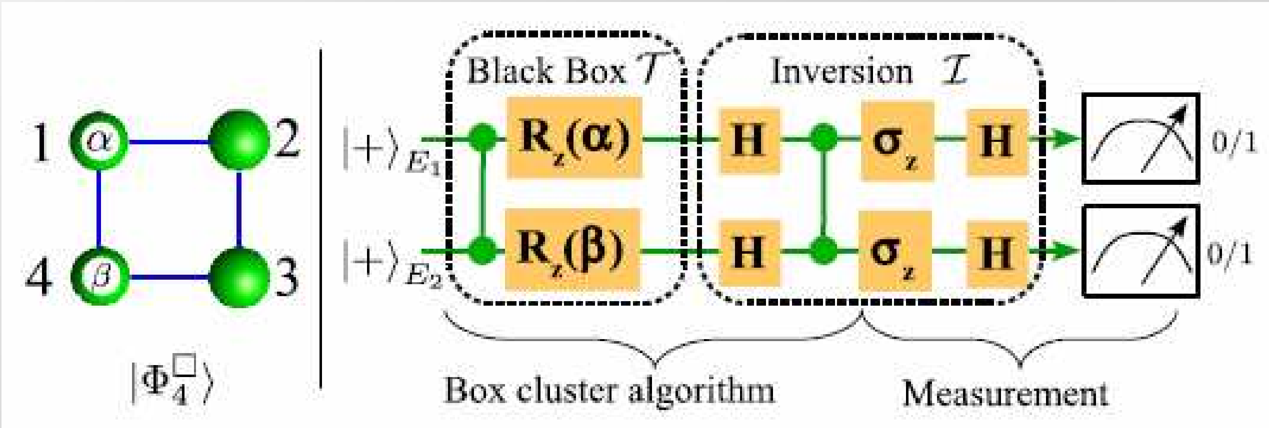
$\mathcal{O}$	α	Control output	$F(s_4 = 0)$	$F(s_4 = 1)$
H	$\pi/2$	$s_1 = 0 \to \|1\rangle_c$	0.965 ± 0.004	0.975 ± 0.004
		$s_1 = 1 \to \|0\rangle_c$	0.972 ± 0.004	0.973 ± 0.004
	$\pi/4$	$s_1 = 0 \to \|1\rangle_c$	0.995 ± 0.008	0.902 ± 0.012
		$s_1 = 1 \to \|0\rangle_c$	0.946 ± 0.010	0.945 ± 0.009

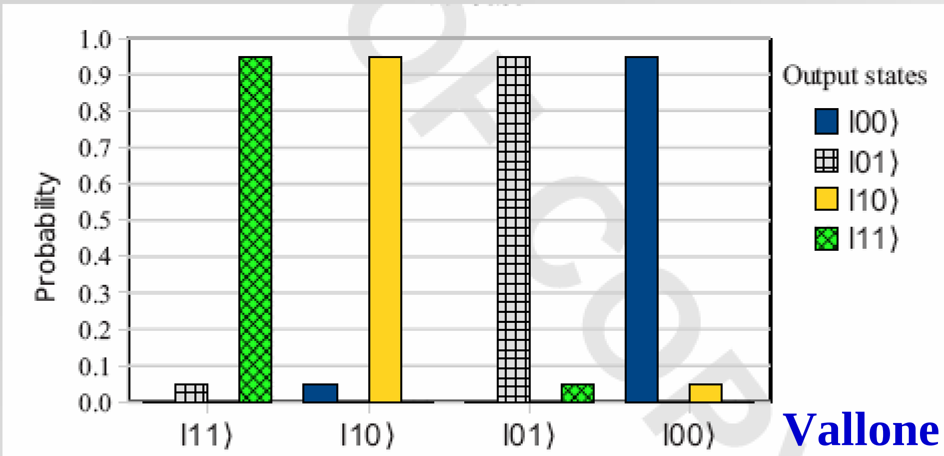
$\mathcal{O}$	α	Control output	$F(s_1 = s_4 = 0)$	$F(s_1 = 0, s_4 = 1)$
$\mathbb{1}$	$\pi/2$	$\|0\rangle_c \equiv \|\ell\rangle_{\mathbf{k_B}}$	0.932 ± 0.004	0.959 ± 0.003
		$\|1\rangle_c = \|r\rangle_{\mathbf{k_B}}$	0.941 ± 0.005	0.940 ± 0.005
	$\pi/4$	$\|0\rangle_c = \|\ell\rangle_{\mathbf{k_B}}$	0.919 ± 0.007	0.932 ± 0.007
		$\|1\rangle_c = \|r\rangle_{\mathbf{k_B}}$	0.878 ± 0.009	0.959 ± 0.006

TABLE II: Experimental fidelity (F) of C-NOT gate output target qubit for different value of α and $\mathcal{O}$.

Grover's search algorithm

Allows to identify the tagged item in a database within 2^M possible solutions (encoded in M qubits).
Right solution found within $\sqrt{2^M}$ steps (classical: $2^M/2$)





Vallone *et al.* PRA, in press

Conclusion and Perspectives

One-Way Quantum Computation with 2-photon 4-qubit cluster states

- ***Low decoherence***
- ***High repetition rates***
- ***High fidelity of the algorithms***

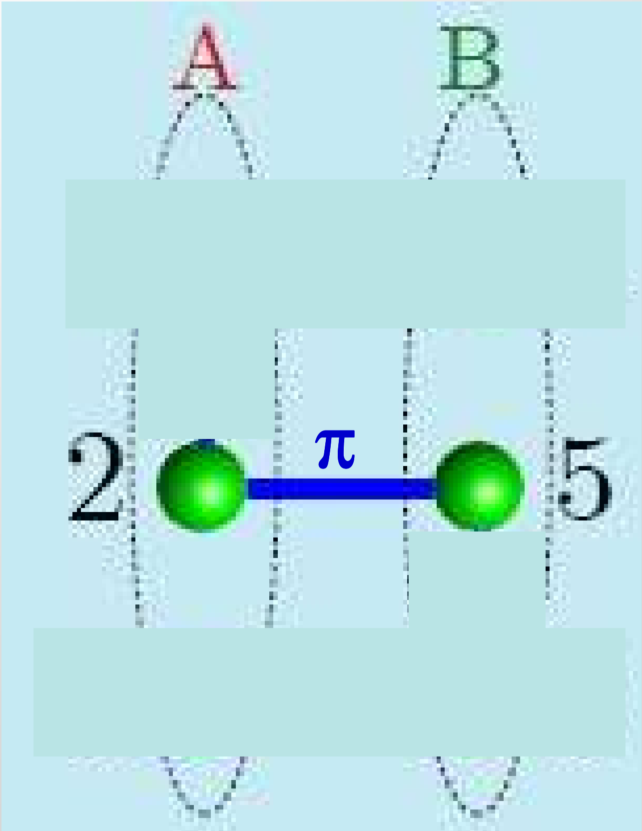
Need to increase the computational power by using more qubits

Different strategies:

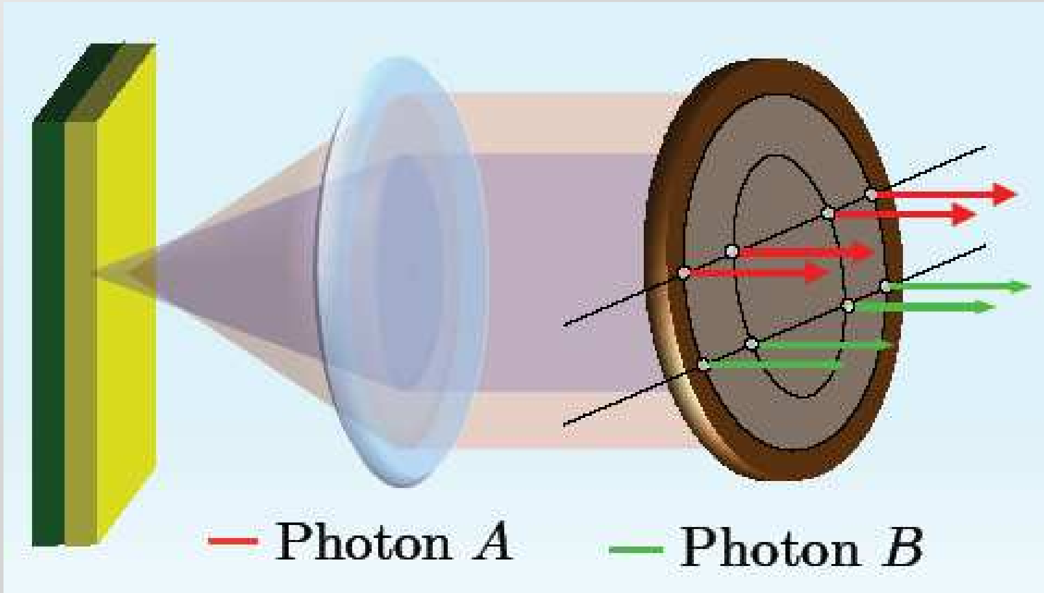
- **Use more degrees of freedom**
- **Use more photons**
- **Hybrid approach (more photons + more degreees of freedom)**

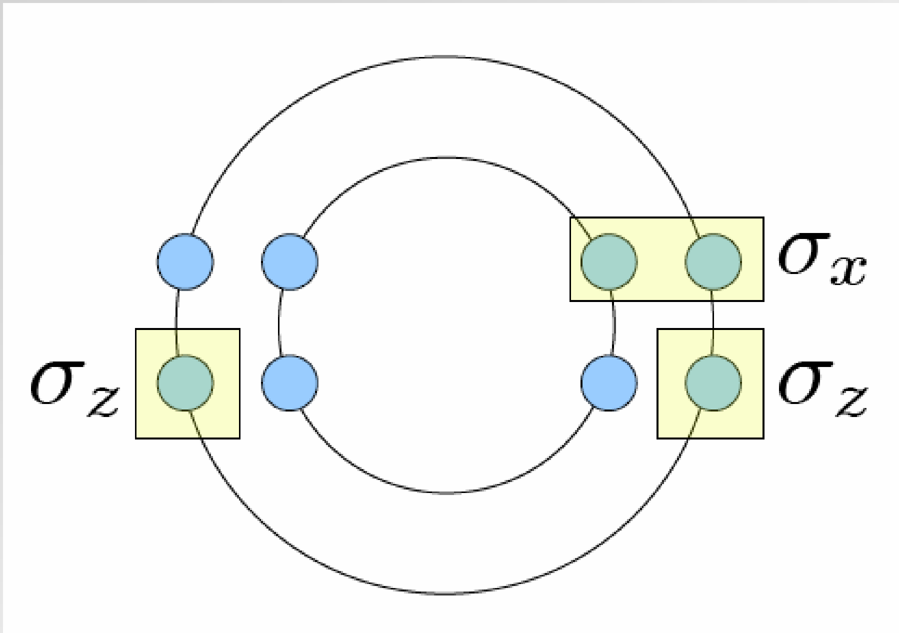
6-qubit cluster state (based on triple entanglement of two photons)

2-crystal geometry



LC6

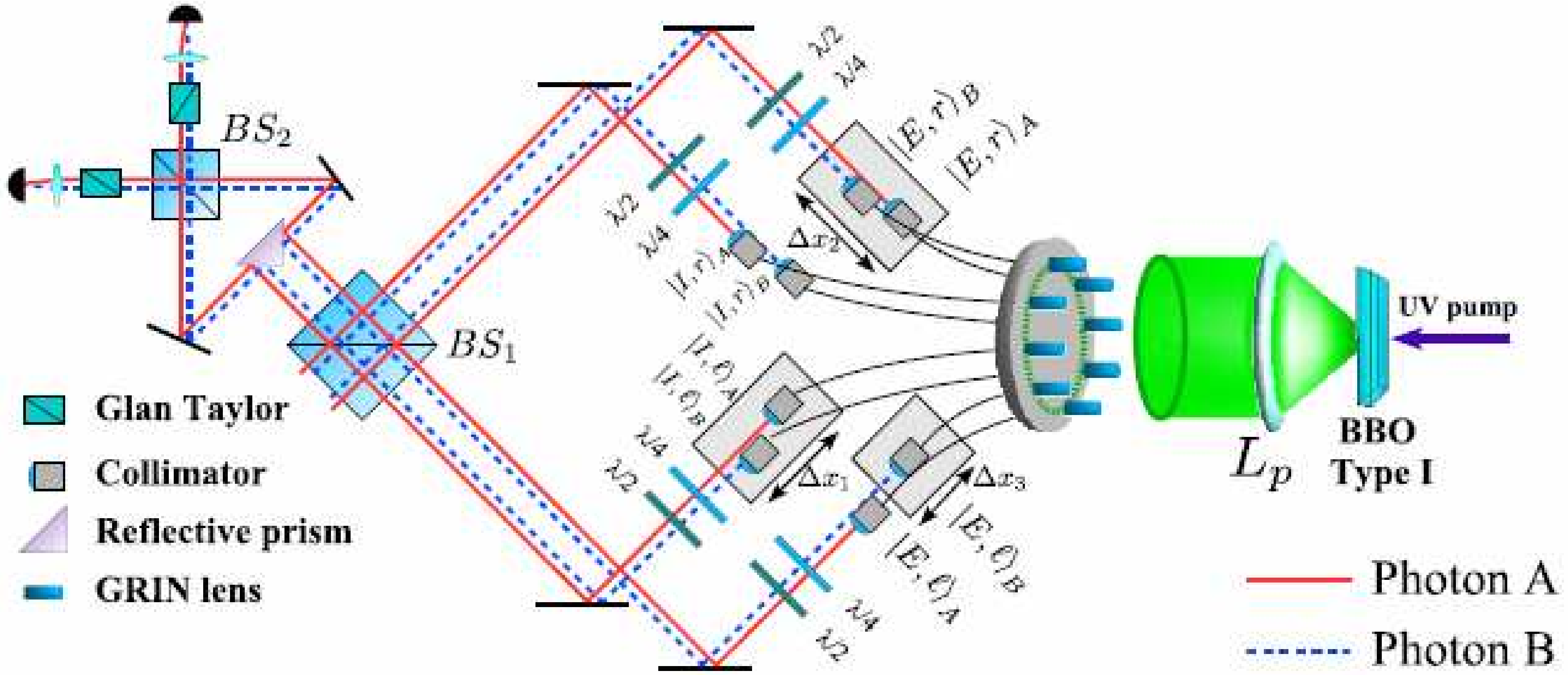




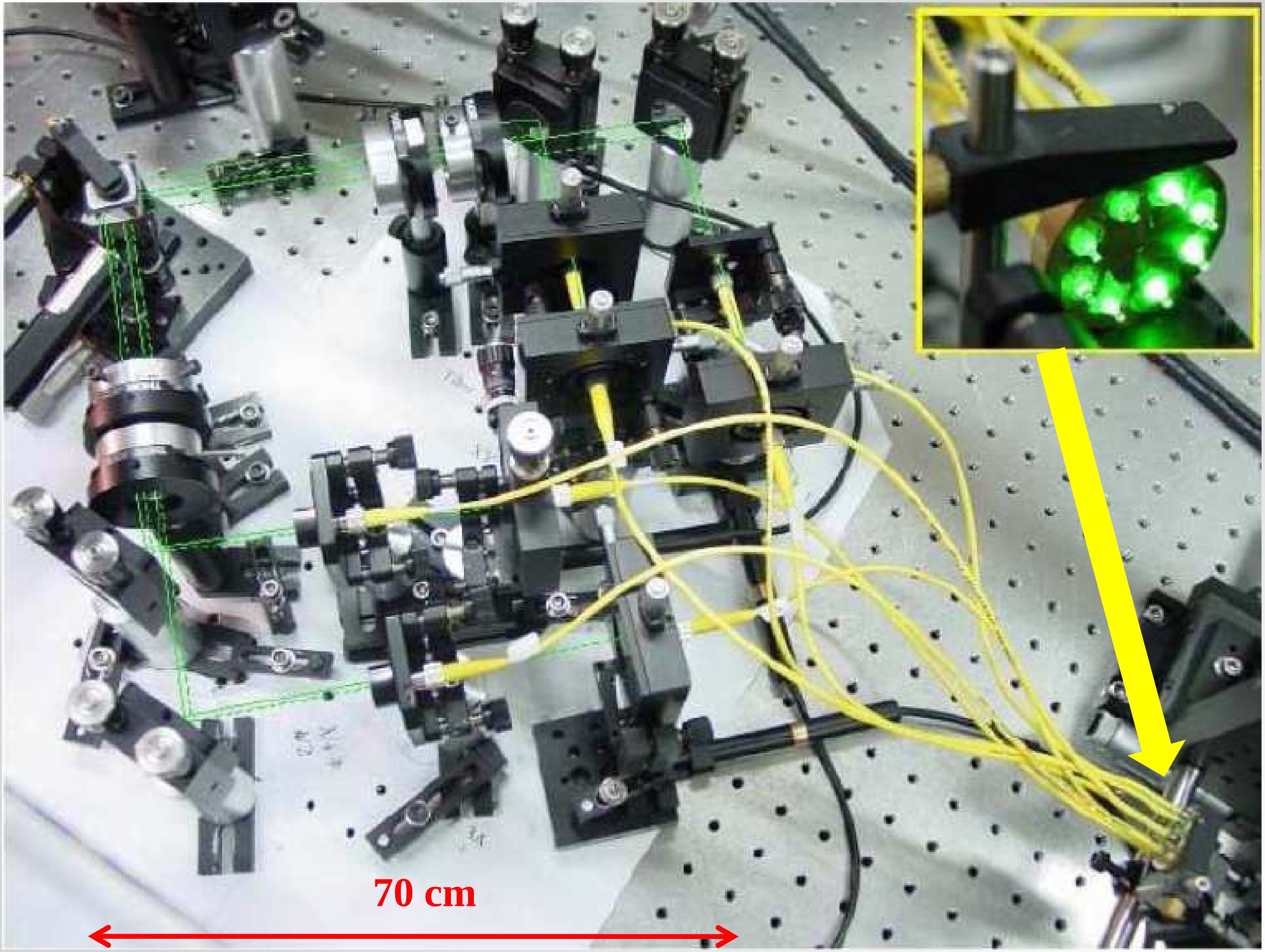
Integrated system of GRIN lenses with single mode optical fibers

Allows efficient coupling of SPDC radiation belonging to many optical modes $\Longrightarrow$ Multipath Entanglement

Measurement setup

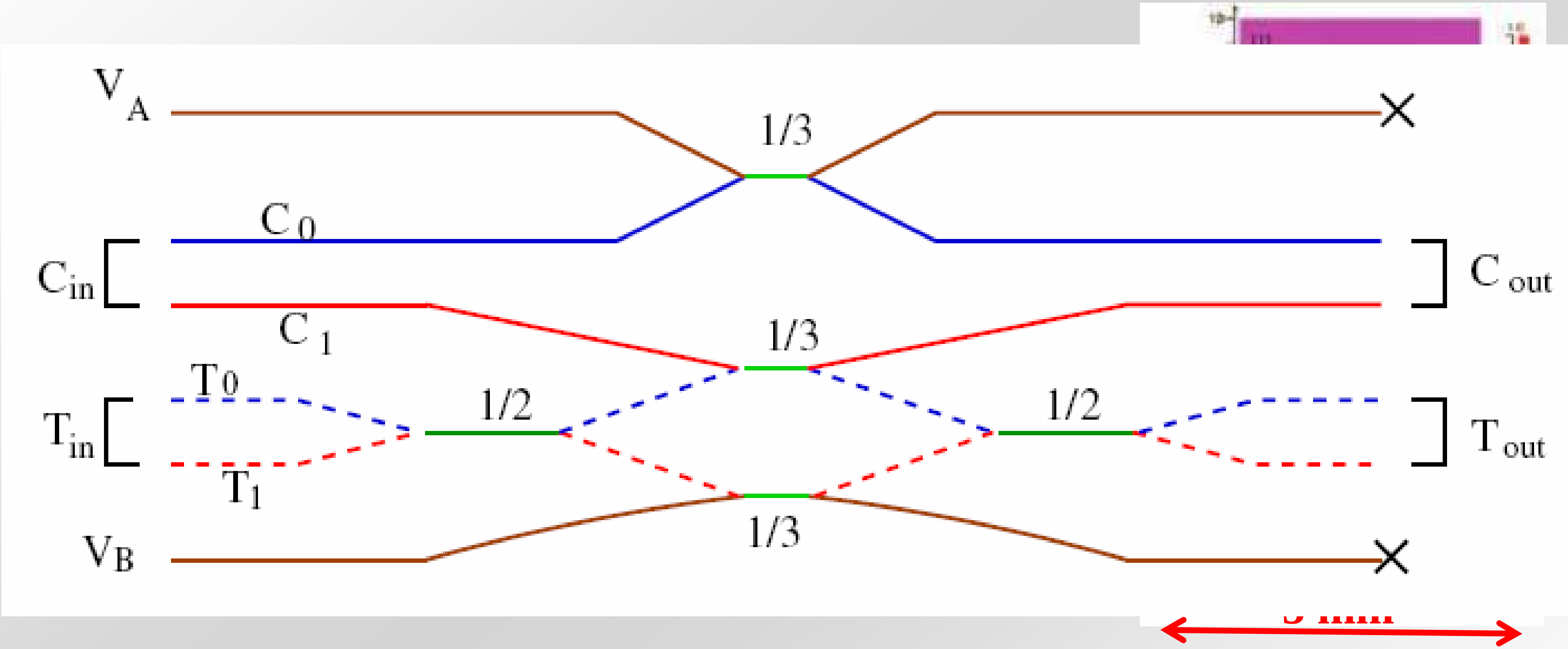


Rossi *et al.* ar-Xiv: quant-ph 08


70 cm

An important result: use of integrated optics

O' Brien, Science '08



Completely integrated C-NOT gate
Future quantum circuit architectures on chips are now possible

What we need more??

- More and more qubits to put in a cluster state (more photons, more degrees of freedom…)
- More efficient and compact sources of entangled photons (to be integrated on waveguide chips)
- New optical tools to manipulate photons (i.e. quantum converters between different degrees of freedom)
- Efficient error corrections

but, in particular,

a REAL, deterministic, high repetition rate source of n-photn Fock states
(in particular single photons $\longrightarrow$ Photon gun)

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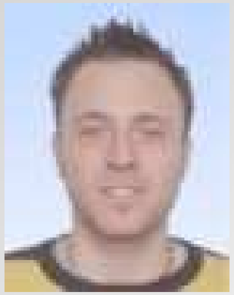

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