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ENERGY HARVESTING TRANSDUCERS - PIEZOELECTRIC

(ICT-ENERGY SUMMER SCHOOL 2016)

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Three Types of Electromechanical Lossless Transduction

1. **Electrodynamic** (also called **electromagnetic** or inductive): motor/generator action is produced by the current in, or the motion of an electric conductor located in a fixed transverse magnetic field (e.g. voice coil speaker)

2. **Piezoeletric**: motor/generator action is produced by the direct and converse piezoelectric effect – dielectric polarization gives rise to elastic strain and vice versa (e.g. tweeter speaker)

3. **Electrostatic**: motor/generator action is produced by variations of the mechanical stress by maintaining a potential difference between two or more electrodes, one of which moves (e.g. condenser microphone)

Credit: This classification and much of the flow from Electromagnetic section is based on the 2013 PowerMEMS presentation by Prof. David Arnold at the University of Florida

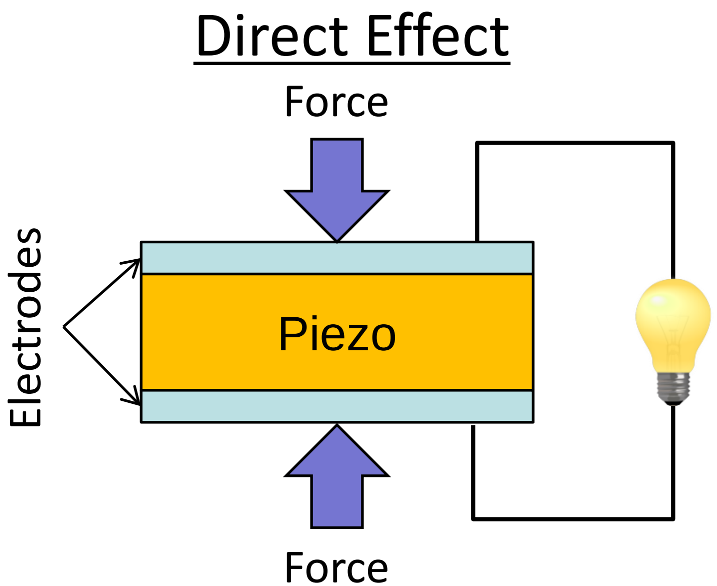
Outline for Short Course

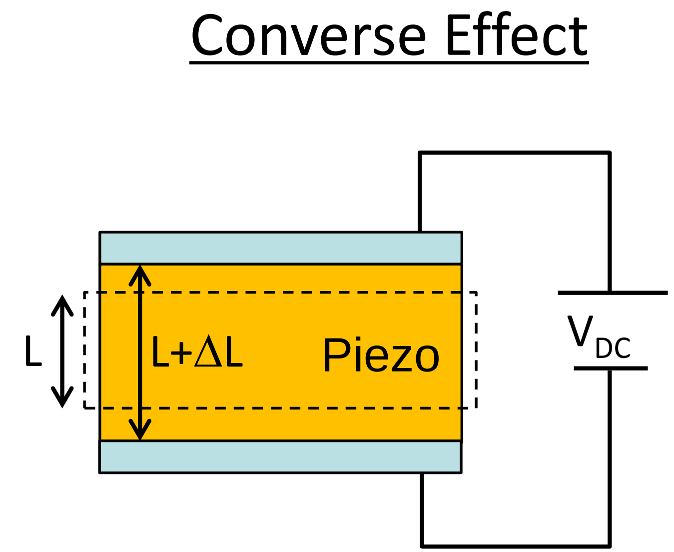
- Introduction and Linear Energy Harvesting
- **Energy Harvesting Transducers**
  - Electromagnetic
  - **Piezoelectric**
  - Electrostatic
- Wideband and Nonlinear Energy Harvesting
- Applications

Outline

- **Fundamental equations of piezoelectricity**
- Piezoelectric energy harvesting (without dynamics)
- Survey of materials
- Dynamics of vibration energy harvesting
- Current research and examples

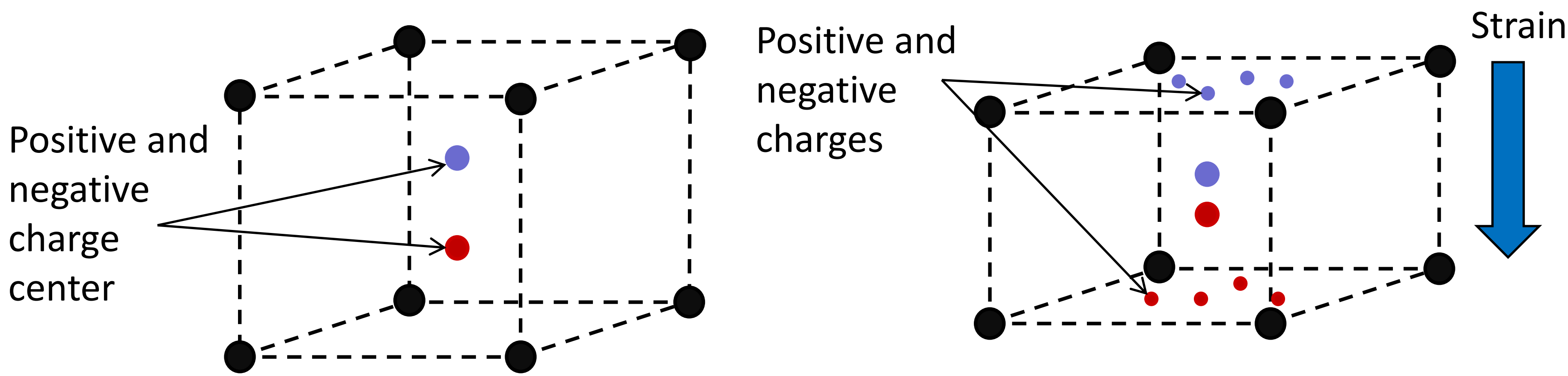
Piezoelectricity





- Piezoelectric materials produce charge from an applied mechanical stress (direct effect), or undergo strain in response to an applied electrical field (converse effect).
- Piezoelectric materials can be crystalline, poly-crystalline, or semi-crystalline. The most common piezoelectric materials for energy harvesting are PZT, AlN, and PVDF (which is a semi-crystalline polymer).
- We are mostly concerned with the direct effect, although the equations apply equally to either effect.
- When mechanical stress is applied charge sites shift, creating a net electric field.

Piezoelectricity



Adapted from Briand, et. al. 2015

- Crystal structure does not have a center of symmetry.
- Under no mechanical force, the system is under equilibrium.
- If material is subjected to mechanical stress or strain, the crystalline structure is deformed, the distance between the charge centers changes.
- In order to keep electrical neutrality, charges appear at the surface of the crystal.

Fundamental Equations

Strain

Compliance

Stress

Piezoelectric strain coefficient

$$\{S\} = [s^E]\{T\} + [d^t]\{E\}$$

$$\{D\} = [d]\{T\} + [\epsilon^T]\{E\}$$

Electrical displacement

Permittivity

Electric field

$[\ ]^E$ – constant electric field (short circuit)

$[\ ]^D$ – constant charge (open circuit)

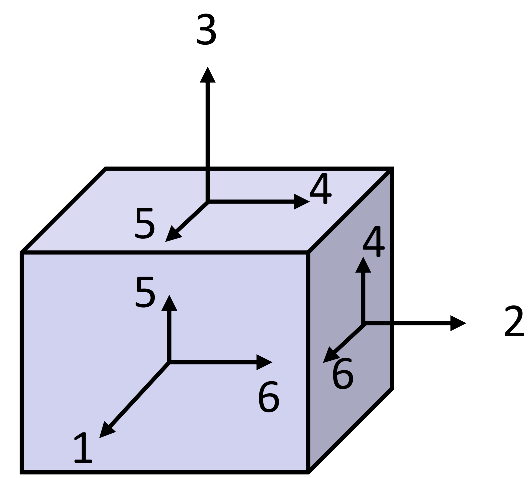
$[\ ]^T$ – constant stress (free condition)

$[\ ]^S$ – constant strain (clamped condition)

- Note, without the piezoelectric strain coefficient we just have Hooke’s law and the equation for a dielectric material.
- The “d” coefficient relates strain to electric field (strain coefficient) and charge to stress (charge coefficient).

Fundamental Equations

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\ s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

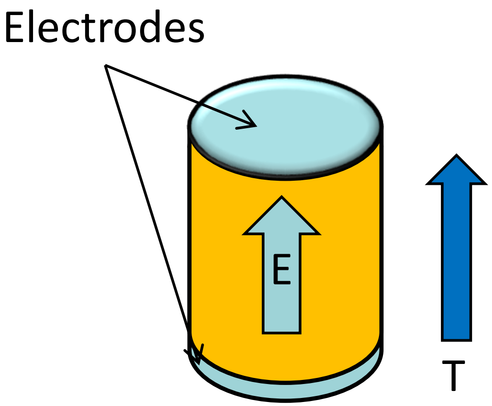


$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

- 4, 5, 6 are the shear stress and strain directions.
- 3 is generally the direction of electrical poling
- In most actual situations, these are reduced to scalar equations due to geometric and constraints and placement of electrodes.

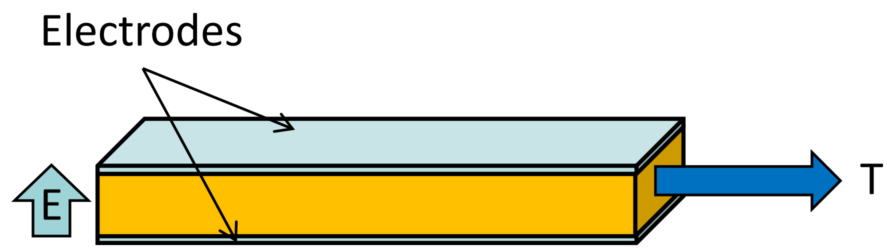
Common Geometries

33 mode



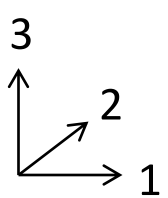
$$S_3 = s_{33}^E T_3 + d_{33} E_3$$
$$D_3 = d_{33} T_3 + \epsilon_{33}^T E_3$$

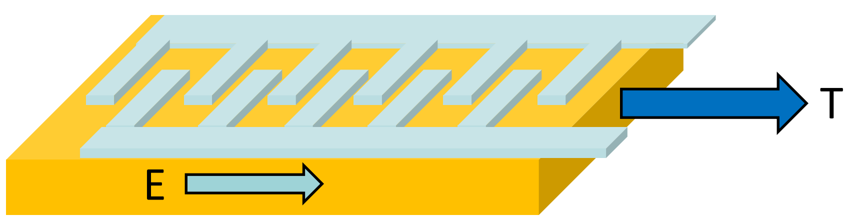
31 mode



$$S_1 = s_{11}^E T_1 + d_{31} E_3$$
$$D_3 = d_{31} T_1 + \epsilon_{33}^T E_3$$

Inter Digitated Electrodes (IDT)





$$S_3 = s_{33}^E T_3 + d_{33} E_3$$
$$D_3 = d_{33} T_3 + \epsilon_{33}^T E_3$$

Alternate Forms of the Equations

$$T = c^E S - eE$$
$$D = eS + \epsilon^S E$$

$c = 1/s$ = stiffness

$e = \left(\frac{\delta D}{\delta S}\right)^E = -\left(\frac{\delta T}{\delta E}\right)^S$ = stress coefficient

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$$S = s^D T + gD$$
$$E = -gT + \beta^T D$$

$\beta = \epsilon^{-1}$

$g = -\left(\frac{\delta E}{\delta \mathrm{T}}\right)^D = \left(\frac{\delta S}{\delta D}\right)^T$ = voltage coefficient

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$$T = c^D S - hD$$
$$E = -hS + \beta^T D$$

$$h = -\left(\frac{\delta E}{\delta S}\right)^D = -\left(\frac{\delta T}{\delta D}\right)^S$$

Note: Subscripts have been dropped here for simplicity.

Piezoelectric Coupling Coefficient

$$W_{con} = W_{OB} - W_{OA}$$

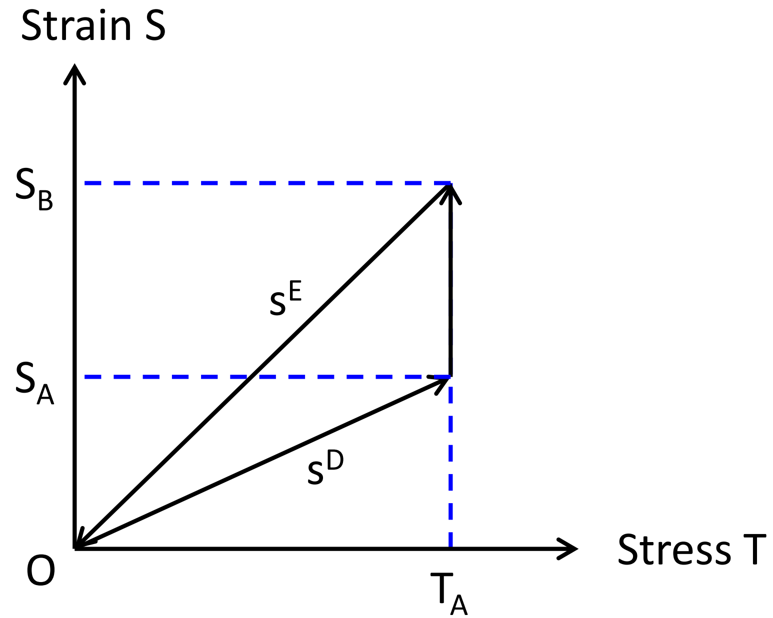
$$k^2 = \frac{W_{con}}{W_{OB}} = 1 - \frac{W_{OA}}{W_{OB}}$$

$$W_{OA} = \frac{1}{2} s^D T_A^2 \quad and \quad W_{OB} = \frac{1}{2} s^E T_A^2$$

$$k^2 = 1 - \frac{s^D}{s^E}$$

Using the constitutive relationships, an equivalent form is:

$$k^2 = \frac{d^2}{s^E \epsilon^T}$$



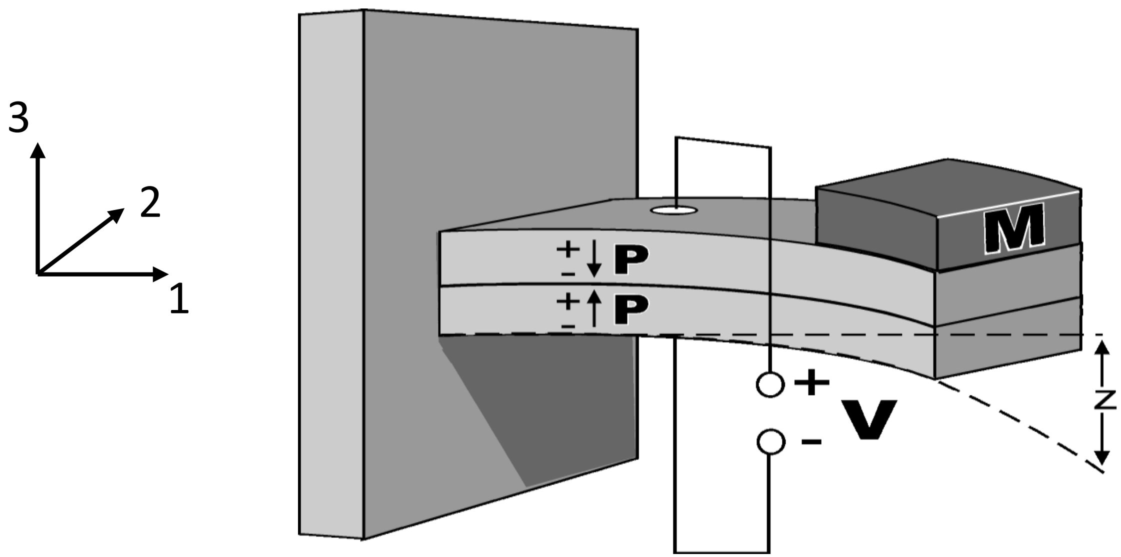
Adapted from Briand, et. al. 2015

- The coupling coefficient squared is the proportion converted energy (e.g. electrical) to the total input (e.g. mechanical) energy
- The coupling coefficient is NOT efficiency. The input energy that is not converted is not lost, but remains stored in the system.
- The coupling coefficient is embedded in the difference between the open and short circuit compliances

Some Relationships Between Coefficients

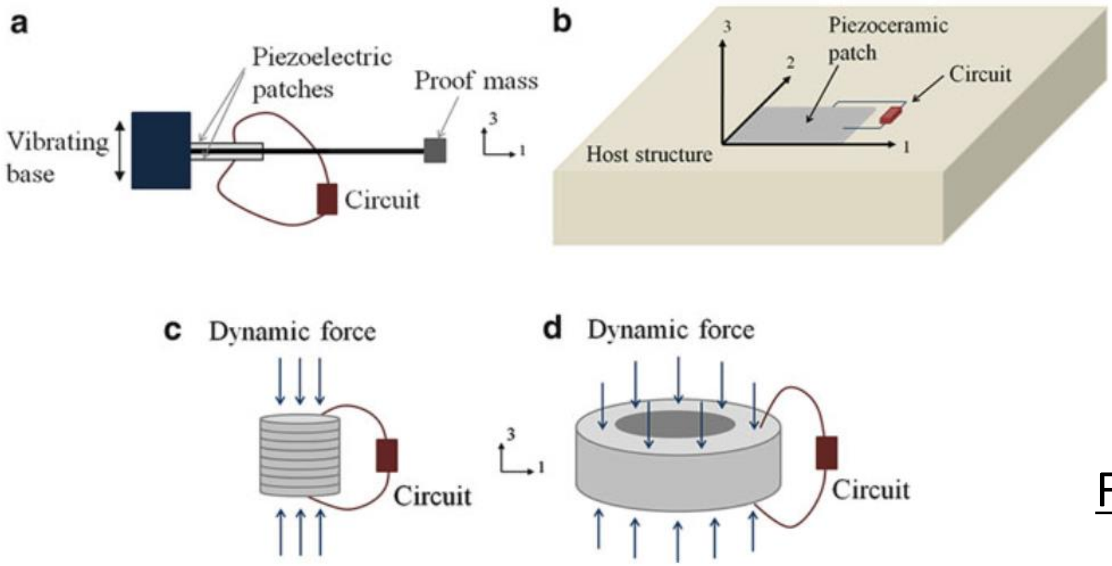
	d	k	g	e
d	$\frac{\text{Strain Developed}}{\text{Applied Electric Field}}$ $\frac{\text{Short Circuit Charge Density}}{\text{Applied Stress}}$	$d_{31} = k_{31}\sqrt{s_{11}^{E}\varepsilon_{33}^{T}}$ $d_{33} = k_{33}\sqrt{s_{33}^{E}\varepsilon_{33}^{T}}$	$d_{31} = g_{31}\varepsilon_{33}^{T}$ $d_{33} = g_{33}\varepsilon_{33}^{T}$	$d_{33} = \frac{e_{33}}{Y_{33}}$
k	$k_{31} = \frac{d_{31}}{\sqrt{s_{11}^{E}\varepsilon_{33}^{T}}}$ $k_{33} = \frac{d_{33}}{\sqrt{s_{33}^{E}\varepsilon_{33}^{T}}}$	$\left(\frac{\text{Mechanical Converted to Elecrical Energy}}{\text{Mechanical Energy Input}}\right)^{1/2}$ $\left(\frac{\text{Electrical Converted to Mechanical Energy}}{\text{Electrical Energy Input}}\right)^{1/2}$	$k_{31} = g_{31}\sqrt{\frac{\varepsilon_{33}^{T}}{s_{11}^{E}}}$ $k_{33} = g_{33}\sqrt{\frac{\varepsilon_{33}^{T}}{s_{33}^{E}}}$	
g	$g_{31} = \frac{d_{31}}{\varepsilon_{33}^{T}}$ $g_{33} = \frac{d_{33}}{\varepsilon_{33}^{T}}$	$g_{31} = k_{31}\sqrt{\frac{s_{11}^{E}}{\varepsilon_{33}^{T}}}$ $g_{33} = k_{33}\sqrt{\frac{s_{33}^{E}}{\varepsilon_{33}^{T}}}$	$\frac{\text{Open Circuit Electric Field}}{\text{Applied Stress}}$ $\frac{\text{Strain Developed}}{\text{Applied Charge Density}}$	

Cantilever Beam – 31 Coupling



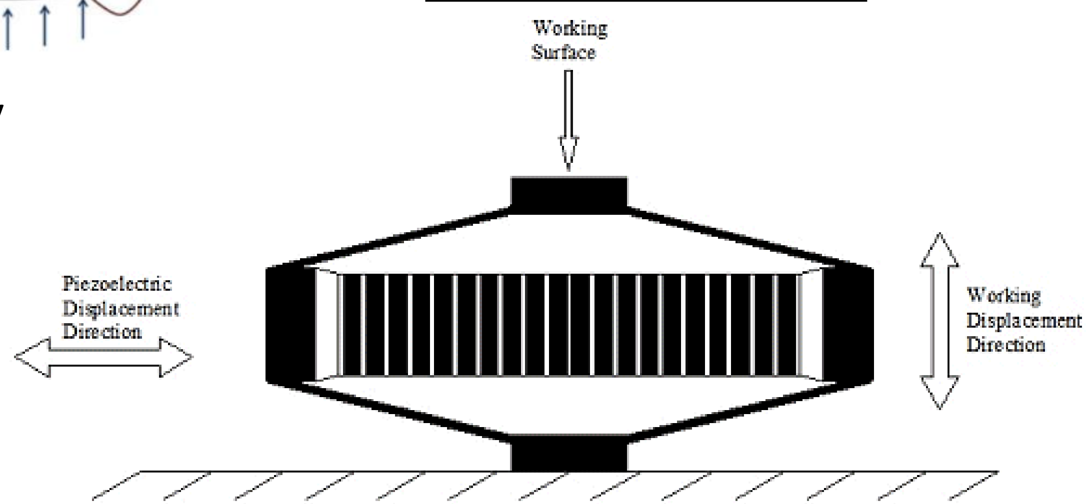
- Although 31 coefficients are generally lower, bending structures are often used because of the ability to produce lower frequency oscillators and generate higher strains

Common Energy Harvesting Structures



Elvin and Erturk, Advances in Energy Harvesting Methods, 2013

Flextensional Device

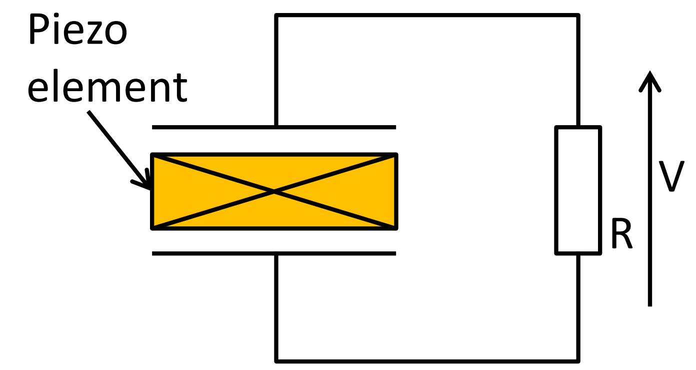


http://www.morpheus.umd.edu/research/systems/piex-flex-actuators.html

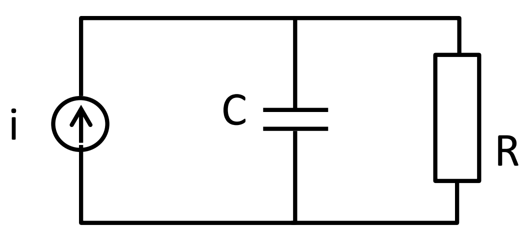
Outline

- Fundamental equations of piezoelectricity
- **Piezoelectric energy harvesting (without dynamics)**
- Survey of materials
- Dynamics of vibration energy harvesting
- Current research and examples

Energy Extraction Cycles – Resistive Load



Adapted from Briand, et. al. 2015



where $C = \frac{\epsilon_{33}^{T} A}{t}$

$t =$ piezo thickness

$A =$ electrode area

$$V = \frac{i}{j\omega C + \frac{1}{R}}$$

$$i = \frac{dQ}{dt} = j\omega D_3 A$$

$$D_3 = d_{3i} S_i$$

Which, after a bunch of algebra, yields:

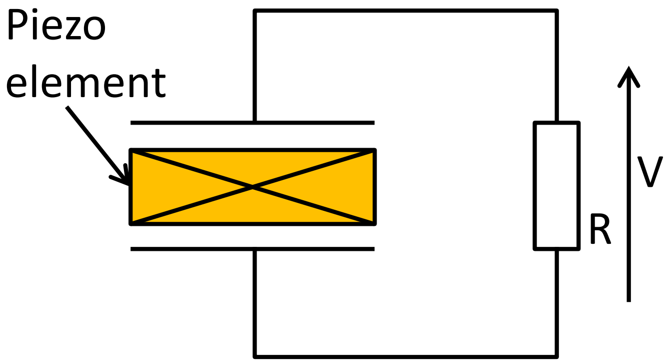
$$|V| = \frac{\omega R d_{3i} c_{ii}^{E} (At)}{\sqrt{(\omega R \epsilon^{T} A)^2 + t^2}} |S|$$

Since $P_{rms} = \frac{|V|^2}{2R}$:

$$P_{rms} = \frac{1}{2} \frac{\omega^2 k_{3i}^2 R c_{ii}^{E} (At)}{\frac{\omega^2 R^2 \epsilon_{33}^{T} A}{t} + \frac{t}{\epsilon_{33}^{T} A}} |S|^2$$

Where we have used: $k_{3i}^2 = \frac{d_{3i}^2}{s_{ii}^{E} \epsilon_{33}^{T}} = \frac{d_{3i}^2 c_{ii}^{E}}{\epsilon_{33}^{T}}$

Energy Extraction Cycles – Resistive Load



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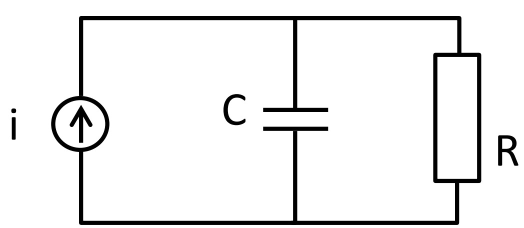

Where $C = \frac{\epsilon_{33}^{T} A}{t}$

$t =$ piezo thickness

$A =$ electrode area

If we differentiate the previous expression with respect to R, we find that:

$$R_{opt} = \frac{t}{\omega \epsilon_{33}^{T} A} = \frac{1}{\omega \mathrm{C}}$$

Substituting in:

$$P_{rms} = \frac{\omega}{4} k_{3i}^{2} c_{ii}^{E} (At) S_i^2$$

$$E_{cyc} = \frac{\pi}{2} k_{3i}^{2} c_{ii}^{E} (At) S_i^2$$

Then the material figure of merit (FOM) is:

$$\boxed{FOM = k_{3i}^{2} c_{ii}^{E} = \frac{d_{3i}^{2} {c_{ii}^{E}}^{2}}{\epsilon_{33}^{T}} = \frac{e_{3i}^{2}}{\epsilon_{33}^{T}}}$$

Increasing Power Output

$$P_{rms} = \frac{\omega}{4} k_{3i}^2 c_{ii}^E (At) S_i^2$$

- Increase the strain level of the piezoelectric material
- Improve material Figure of Merit
- Increase volume of material under strain (i.e. thicker piezoelectric films)
- Increase frequency
  - Note usually you have no control over the excitation frequency
  - In quasi-static applications, increasing frequency can result in more power output
  - In base driven vibration applications, frequency up conversion may have practical benefits, but won't fundamentally result in high power output
- Improve energy extraction circuits

- There is ongoing research addressing each of these issues

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- Fundamental equations of piezoelectricity
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- **Survey of materials**
- Dynamics of vibration energy harvesting
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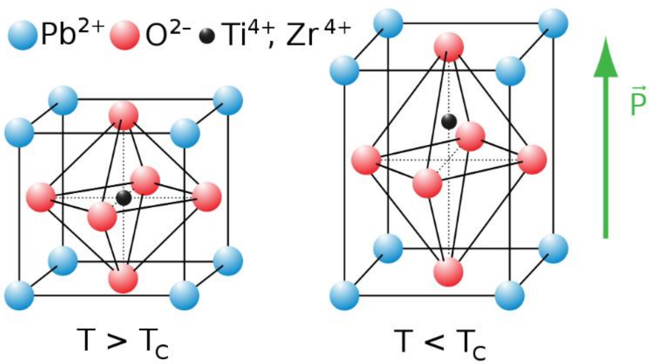
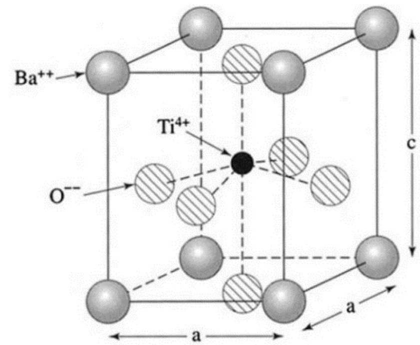
Perovskites



Pb(Zr,Ti)O_3 (PZT)



$BaTiO_3$ (BTO)

Pb(Mg,Nb)O_3-$PbTiO_3$ (PMN-PT)
Single Crystal
TRS Ceramics

- Best performing **piezoelectric** materials so far
- Exhibit ABO_3 structure
- Ferroelectric – must be poled and lose piezoelectricity above the Curie temperature
- Best materials contain lead

Perovskites – Material Comparison

Material	d (10^{-12} m/V)	c (10^{9} N/m^2)	ε_{rel}	k	FOM (d^2c^2/ε_{rel})
PZT-5A[1]	d_{33} = 390 d_{31} = -190	c^E_{11}= 66 c^E_{33}= 52	1800	k_{33} = 0.72 k_{31} = 0.35	FOM_{33}=0.23 FOM_{31}=0.09
PZT-5H[1]	d_{33} = 650 d_{31} = -320	c^E_{11}= 62 c^E_{33}= 50	3800	k_{33} = 0.75 k_{31} = 0.44	FOM_{33}=0.28 FOM_{31}=0.10
PMN-PT[2]	d_{33} = 2820 d_{31} = -1330	c^E_{11}= 11.5 c^E_{33}= 10	8200		FOM_{33}=0.10 FOM_{31}=0.028
PMN-32PT[3]	d_{33} = 2000 d_{31} = -920	c^E_{11}= 20 c^E_{33}= 20	4950	k_{33} = 0.95 k_{31} = 0.78	FOM_{33}=0.32 FOM_{31}=0.07

- Bulk material properties
- Thinfilms are generally lower

[1] www.piezo.com

[2] Cao et. al. J. Appl. Phys. Vol. 96, No. 1, 2004

[3] Briand et. al. 2015

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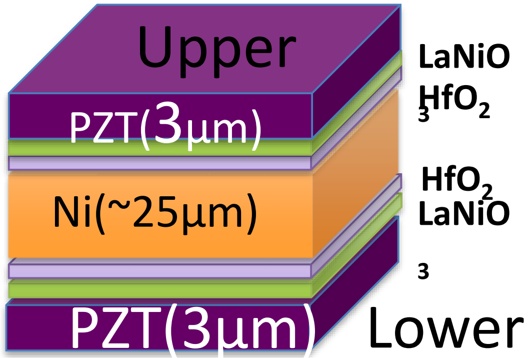
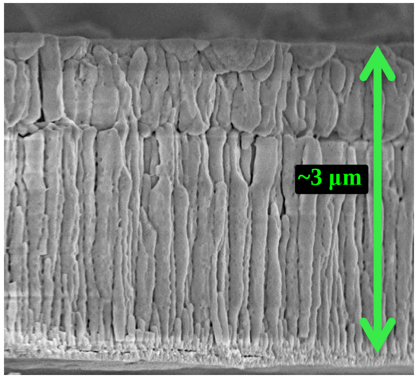
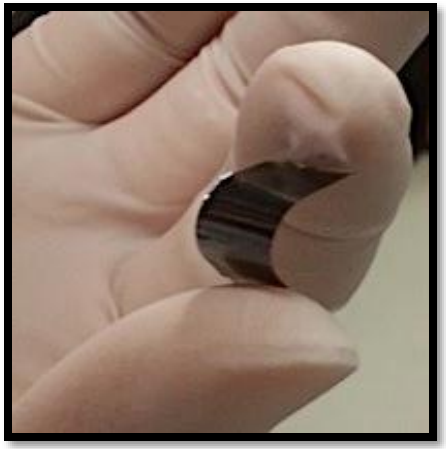
Advances In Thinfilm PZT

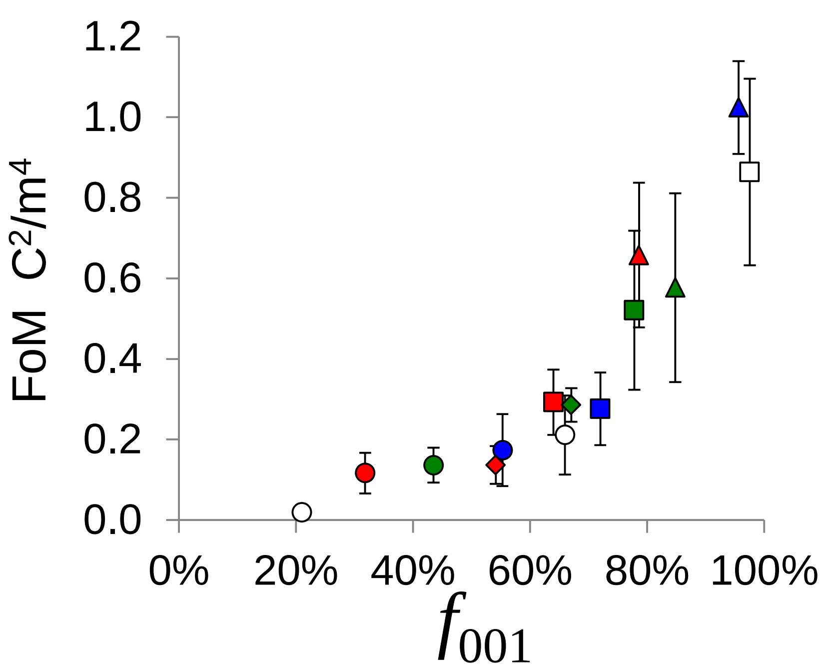




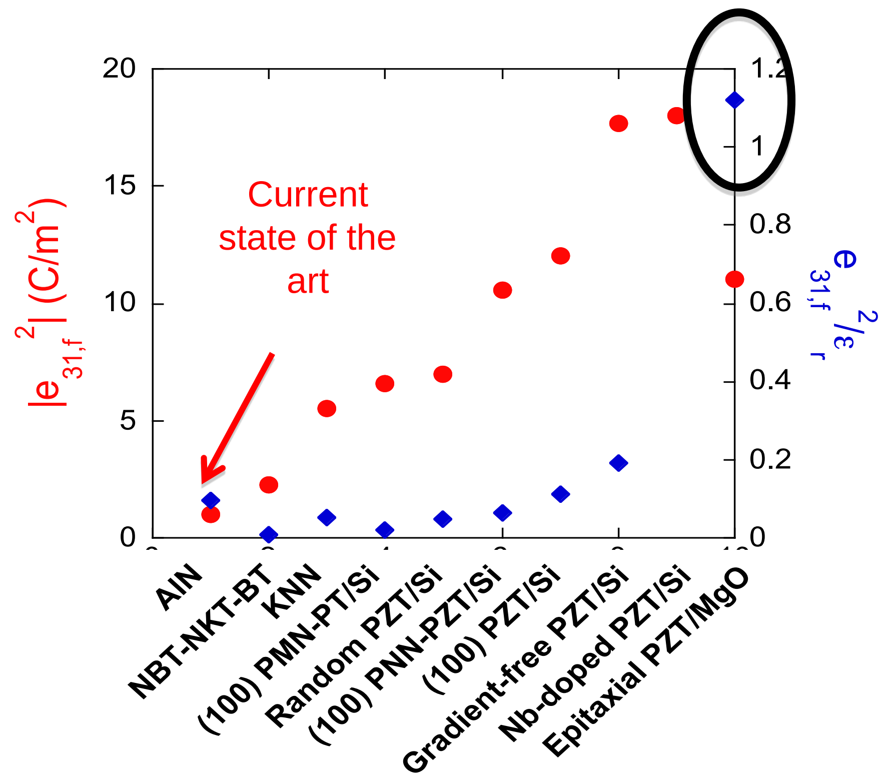


- Susan Trolier-McKinstry's group at Penn State has developed very high quality PZT thinfilms fabricated on nickel or silicon substrates

Advances In Thinfilm PZT

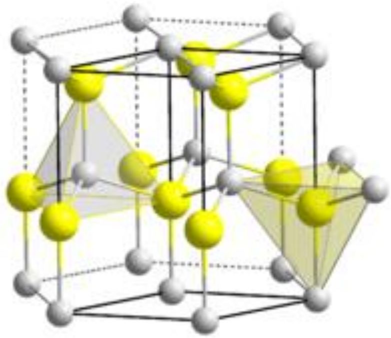


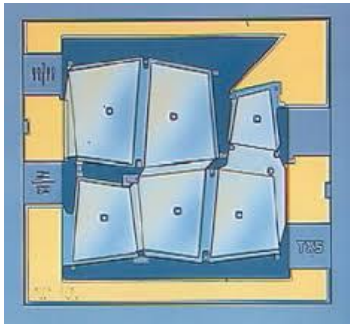
Yeager, Funakubo, Trolier-McKinstry et al., JAP, 2012



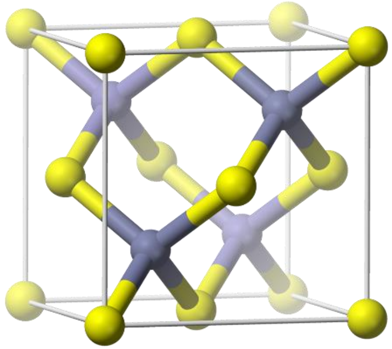
S. H. Baek et. al., Science 2011

Wurtzites





Aluminum Nitride (AlN)



Zinc Oxide (ZnO)

- Exhibit permanent polarization
  - Can operate to very high temperatures
  - Do not need to be poled
- Wide range of material properties from different groups producing thinfilms

AlN – Material Comparison

Material	d (10^{-12} m/V)	c (10^9 N/m^2)	ε_{rel}	k	FOM (d^2c^2/ε_{rel})
AlN[1]	d_{33} = 5 d_{31} = -2.5	c^E_{11}= 300 c^E_{33}= 300	10	k_{33} = 0.07	FOM_{33}=0.23 FOM_{31}=0.06
AlN[2]	d_{31} = -147	c^E_{11}= 395 c^E_{33}= 395	9.5	k_{31} = 0.1	FOM_{31} =0.035
AlN[3]					FOM_{31} =0.1

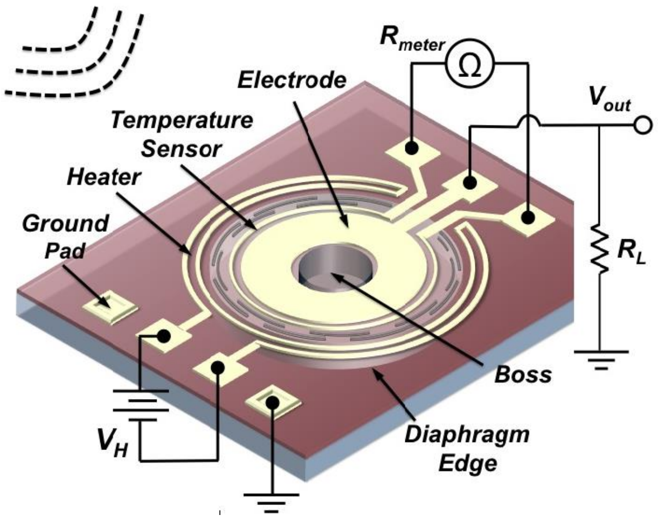
- All thinfilm properties

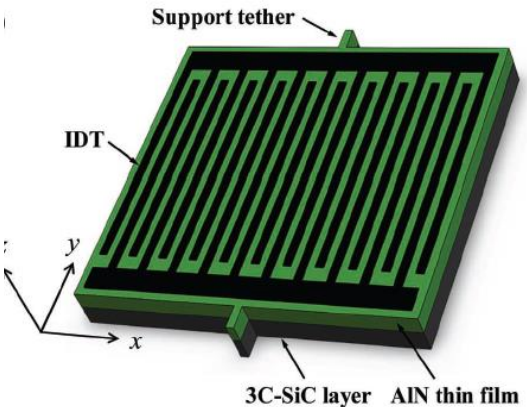
[1] Defay, *Integration of Ferroelectric and Piezoelectric Thinfilms* (2011)

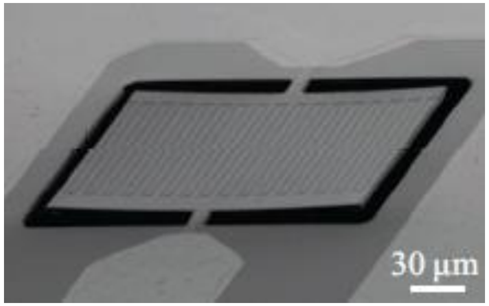
[2] Lin et. al. J. Advanced Materials (2012)

[3] Andosca et al., Sensors and Actuators A, 178 (2012) 76

Harsh Environment Applications





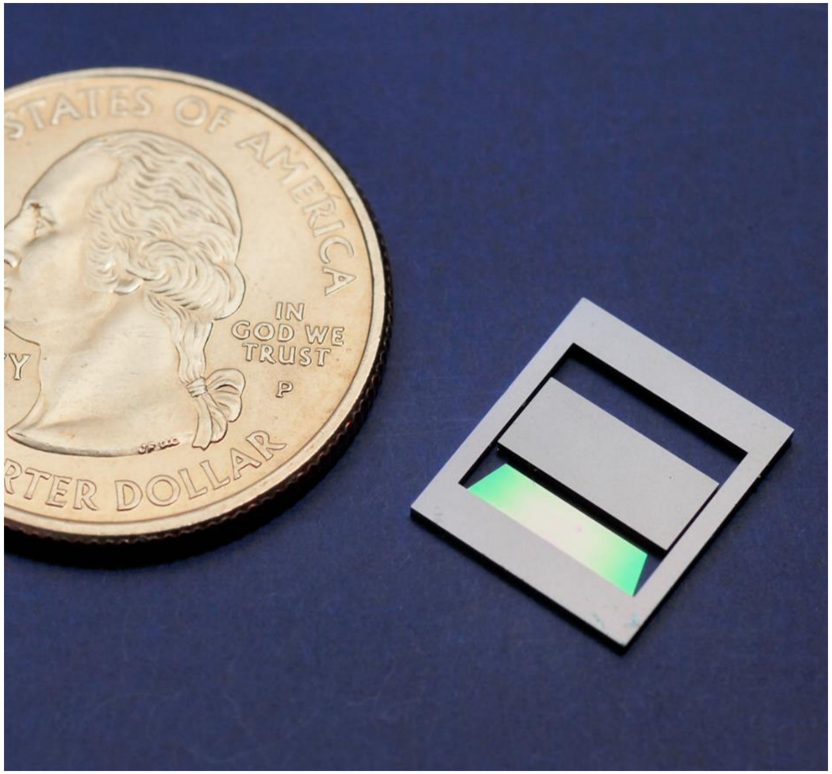


High temperature Energy Harvesters
Lai et. al. Transducers 2013

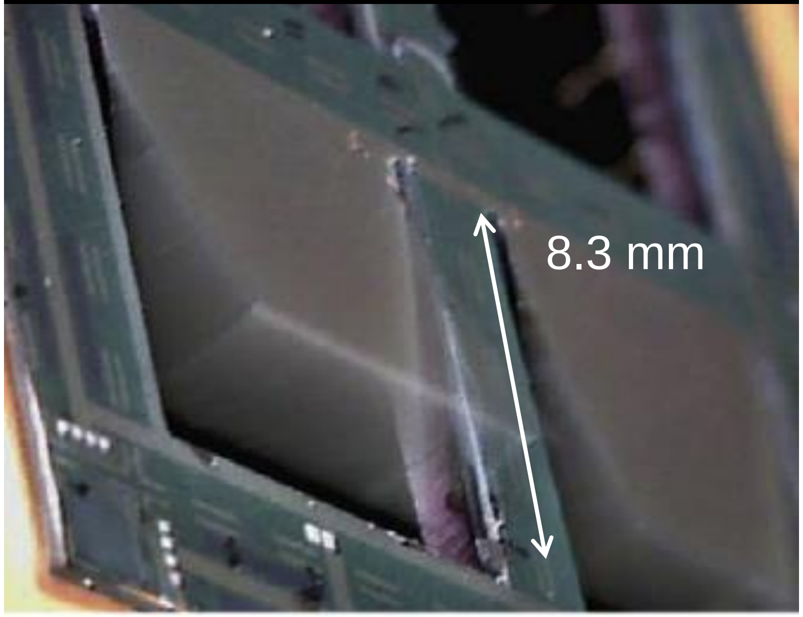
High temperature GHz Resonator
Li et. al., Adv. Mater. 2012, 24, 2722-2727

- Material is non-ferroelectric
- Maintains piezoelectric properties up to at least several hundred degrees C

MicroGen Systems



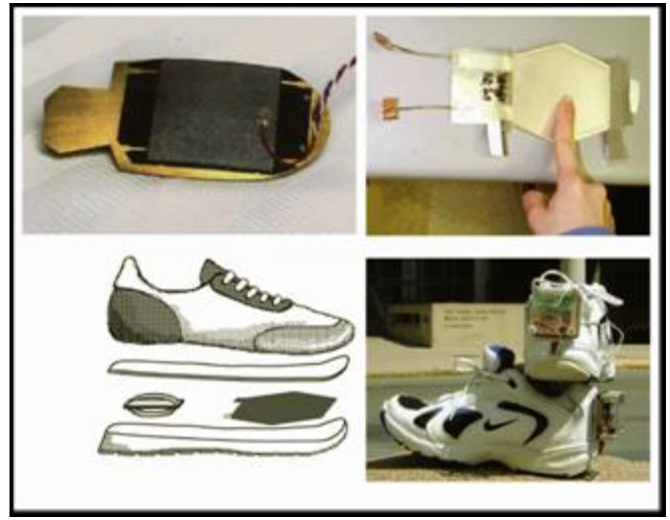
Andosca et al., Sensors and Actuators A, 178 **(2012)** 76



https://www.microgensystems.com/

PVDF

Measurement Specialties Inc.



Paradiso and Starner, *IEEE Pervasive Computing*, 2005.

Material	d (10^{-12} m/V)	c (10^{9} N/m^2)	ε_{rel}	k	FOM (d^2c^2/ε_{rel})
PVDF[1]	d_{33} = -30 d_{31} = 22	c^E_{11}= 8.3 c^E_{33}= 8.3	10		FOM_{33}=0.006 FOM_{31}=0.003

- Very poor material figure of merit
- Useful because of cost, flexibility, and robustness

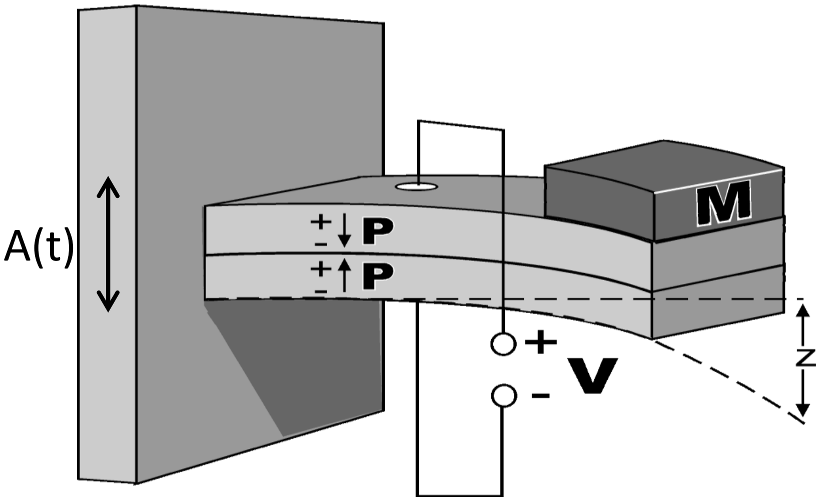
Other Notes

- Concern over the lead in PZT has led to development of other lead-free piezoelectric materials such as $(K,Na)NbO_3$ (KNN)
- For MEMS devices, PZT is commonly fabricated with either a sol-gel process or by sputtering
  - Max thickness is generally 2 um or less
- AlN is usually fabricated by sputtering
  - Max thickness is generally 2 um or less
  - Multi-user AlN process form MEMSCAP (PiezoMUMPs) http://www.memscap.com/products/mumps/piezomumps

Outline

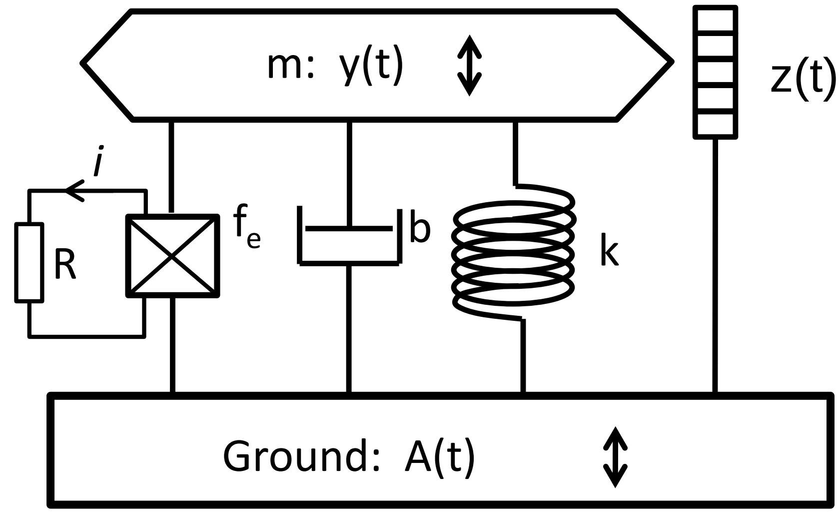
- Fundamental equations of piezoelectricity
- Piezoelectric energy harvesting (without dynamics)
- Survey of materials
- **Dynamics of vibration energy harvesting**
- Current research and examples

Vibration Energy Harvesters (VEHs)



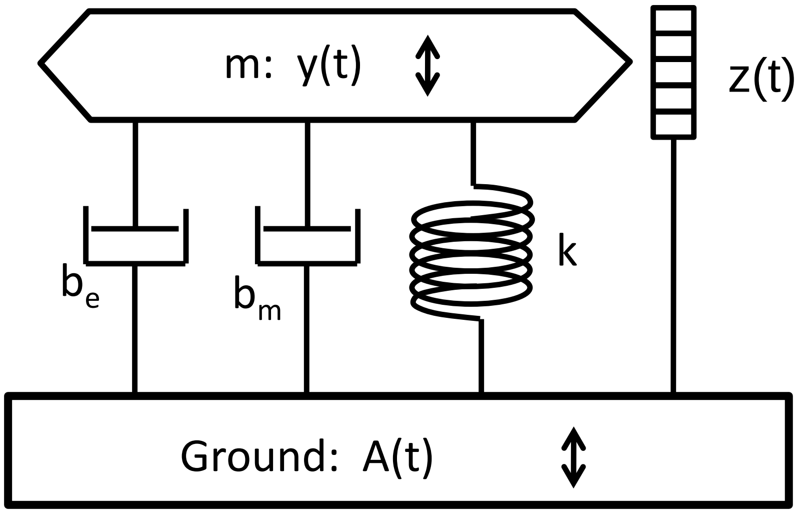
Roundy & Wright, SMS, 2004

- Piezoelectric bimorph beam VEH
- Base (or wall) is driven by vibration with acceleration A(t)
- There is a proof mass at the end of the beam, which flexes causing a 3-1 mode piezoelectric transducer



- Lumped parameter model of the piezoelectric beam VEH
- Mass is the equivalent mass (actual mass and inertial effects of beams)
- Piezo element creates an electrically induced force (f_e) on the mechanical oscillator as well as generating current through a load circuit

VDRG Vibration Energy Harvester Model



- b_e is the electrically induced damping coefficient
- Power dissipated through b_e is the power extracted by the load circuit
- This model has been shown to produce the upper bound on extractable power from in input dominated by a single, stable frequency

$$Z(j\omega) = \frac{1}{(1-r^2)+j2\zeta r}\frac{A}{\omega_n^2} \quad \text{where } r = \frac{\omega}{\omega_n}$$

$$|Z(j\omega)| = \frac{1}{[(1-r^2)^2+(2\zeta r)^2]^{\frac{1}{2}}}\frac{A}{\omega_n^2}$$

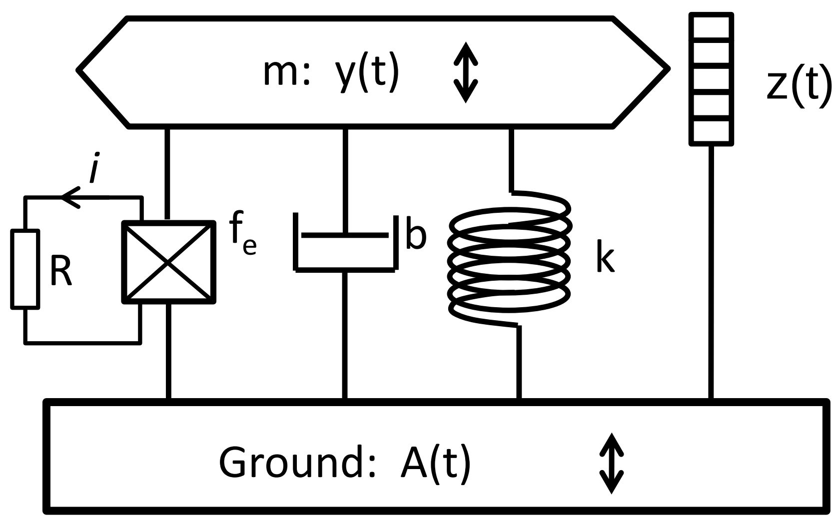
$$P_{rms} = \frac{1}{2}b_e\omega^2|Z(j\omega)|^2$$

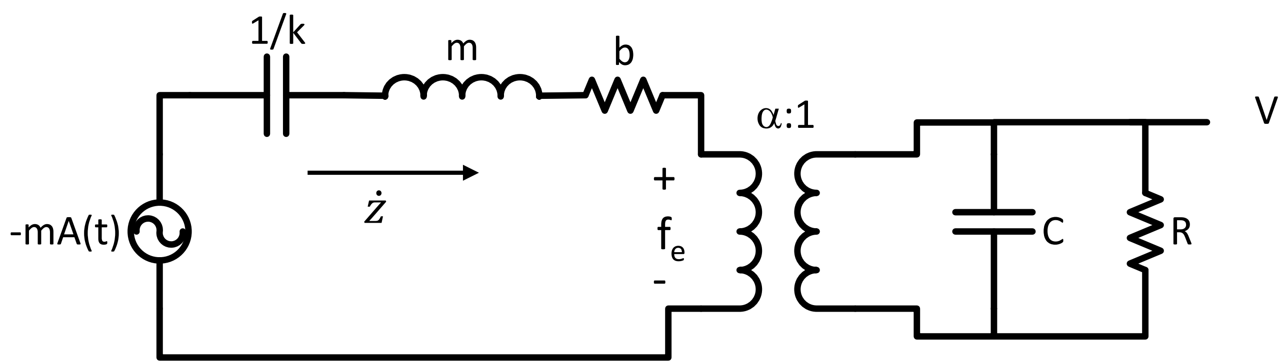
$$P_{rms} = \frac{m\zeta_e r^3 A^2}{\omega[(1-r^2)^2+(2\zeta r)^2]}$$

where $\zeta_e = \frac{b_e}{2m\omega_n}$ and $\zeta = \zeta_e + \zeta_m$

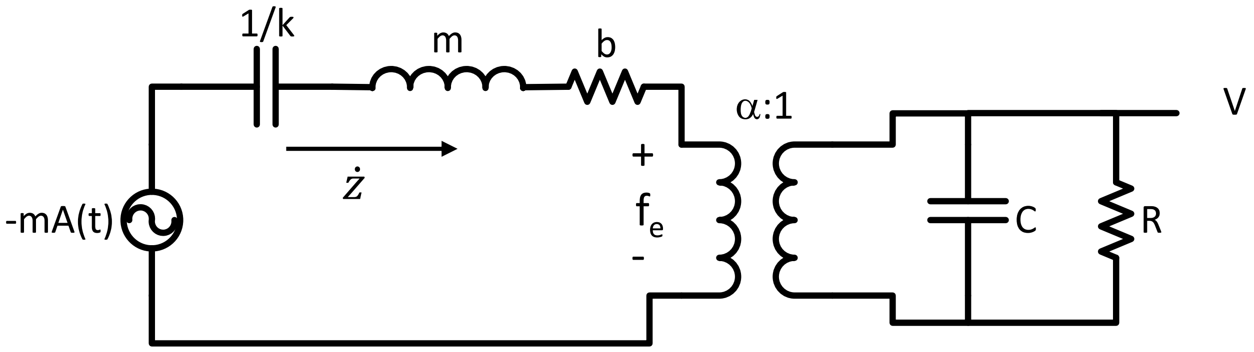
Mitcheson et. al., JMEMS 2004
Halvorsen et. al. J. Phys. Conf. Series, 2013
Heit & Roundy, En. Harv. and Sys., 2015

Adding the Electrical States





Adding the Electrical States



<u>Governing Equations</u>

$$m\ddot{z} + b\dot{z} + kz + \alpha V = -mA$$

$$C\dot{V} + \frac{1}{R}V = \alpha\dot{z}$$

where α is the electromechanical force factor

Note, this is a three state system: $z, \dot{z}, V$

<u>Power Output</u>

$$P_{rms} = \frac{1}{2}\frac{|V|^2}{R}$$

Force Factor α (Cantilever Bimorph)

Transformer Equations

$f_e = \alpha V$

$i = \alpha \dot{z}$

From piezo relationships

$S_1 = s_1^E T_1 + d_{31} E_3$

Assuming a clamped condition:

$T_1 = -c_1^E d_{31} E_3$

Get relationship between f_e and V

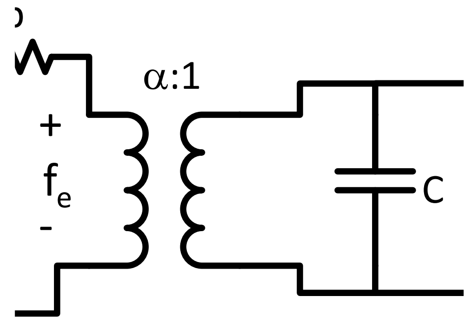
$V = E_3 t$

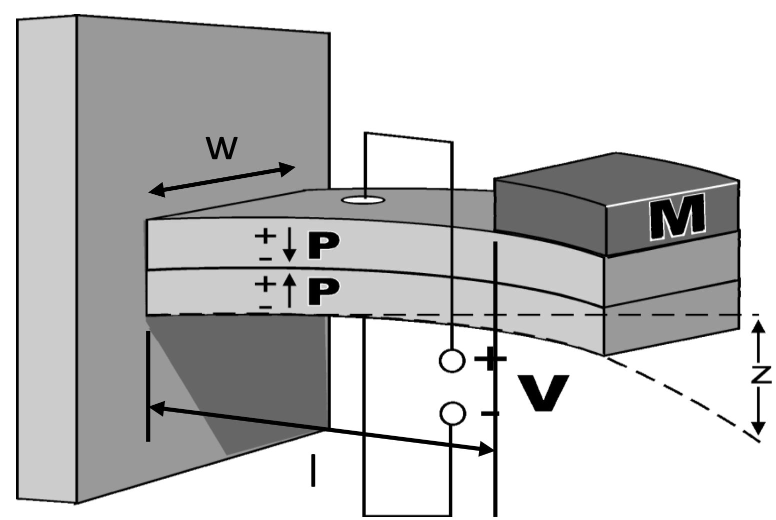
t is the piezo thickness

$f_e = \frac{2wt^2}{3l} T_1$ from beam mechanics

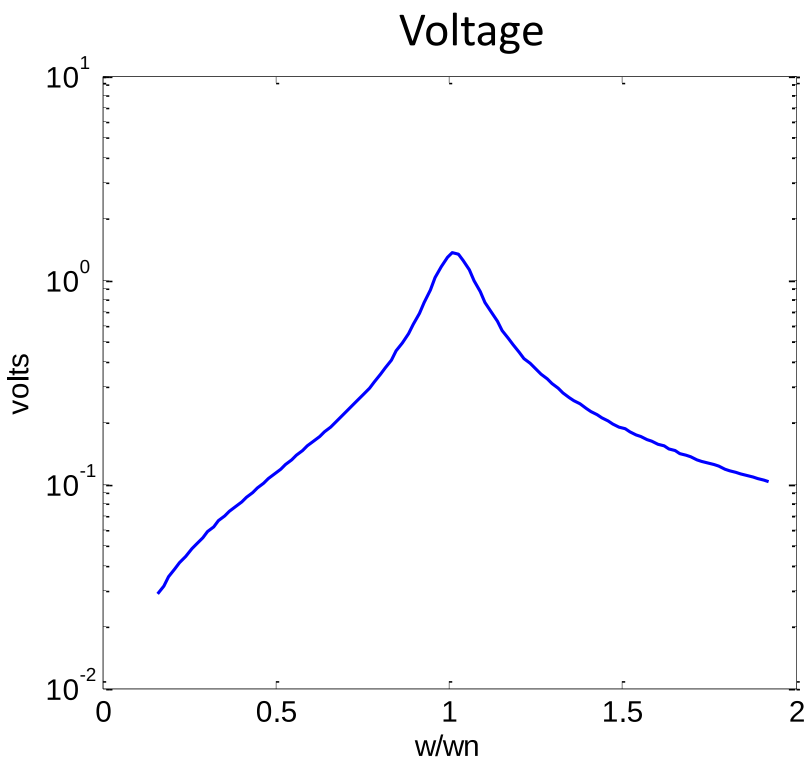
$f_e = -\frac{2wtc_1^E d_{31}}{3l} V$

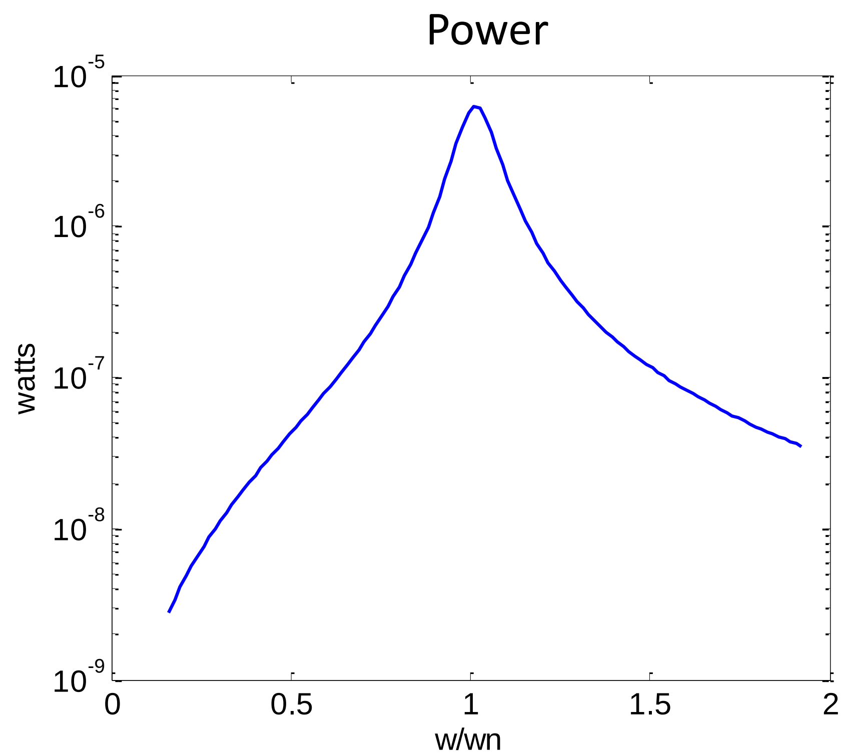
$$\boxed{\alpha = -\frac{2wtc_1^E d_{31}}{3l}}$$





Power and Voltage vs. Frequency





- For PZT-5A bimorph cantilever
  - w = 5mm, l = 20mm, t = 0.125 mm (one layer)
  - A = 1 m/s², m = 1 gram, ζ = 0.025, R = 100 kΩ

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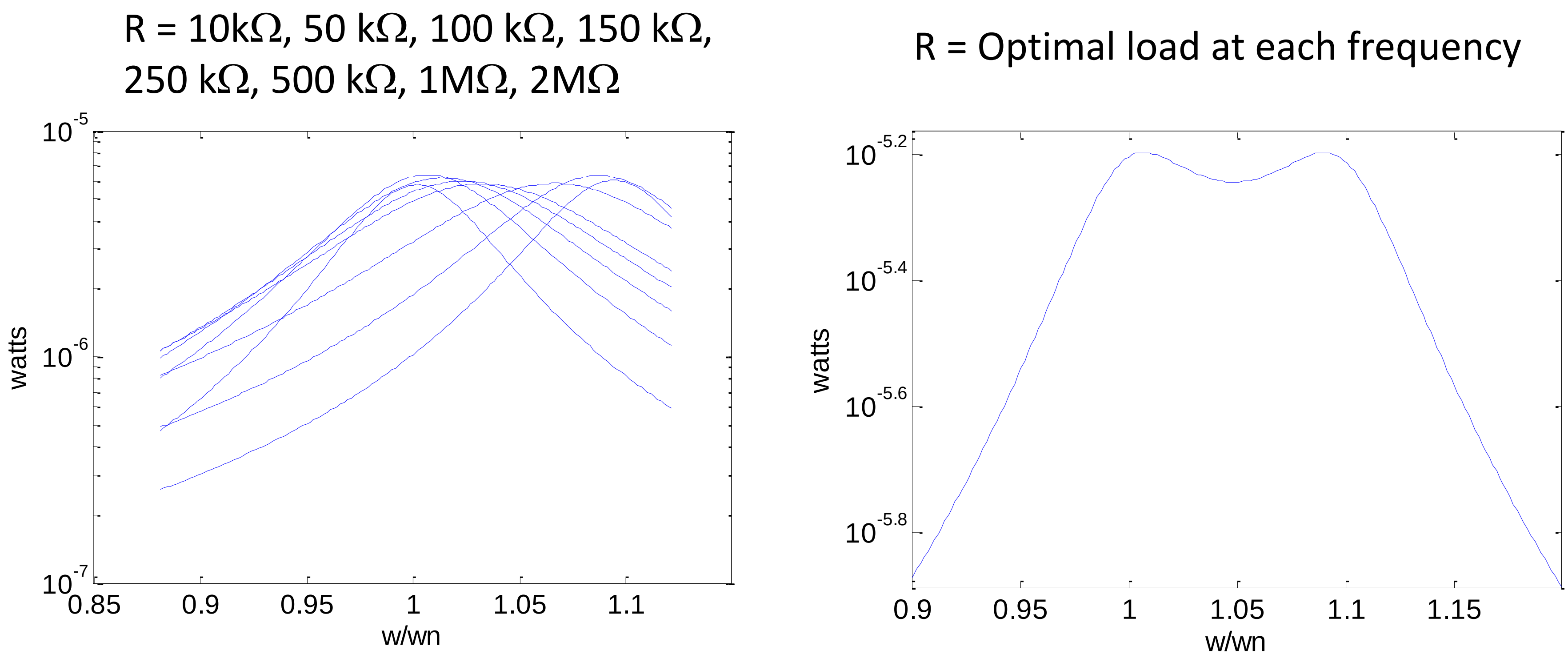
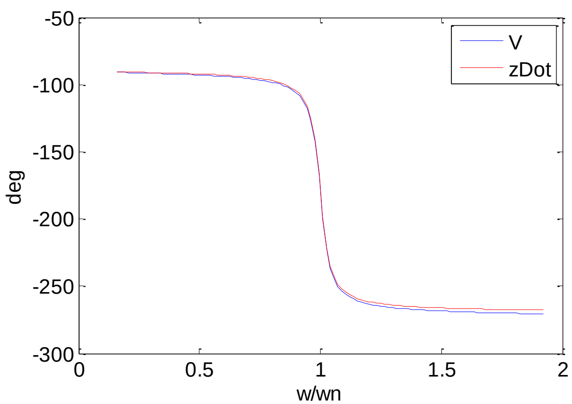
Power vs. Frequency for Different Loads

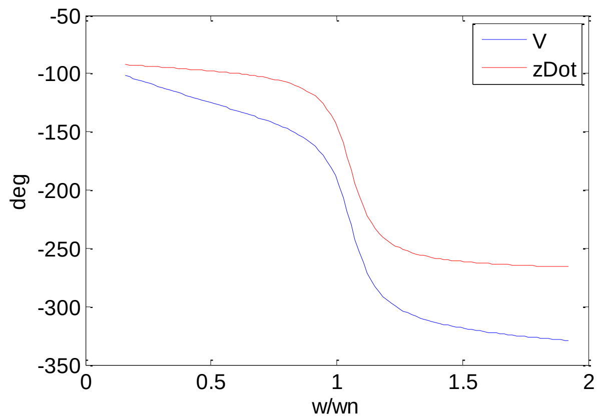


- Resonance frequency shifts from short circuit to open circuit frequencies as resistance moves from low to high
- Highlights ability to shift system resonances with active load circuitry (not part of this lecture).

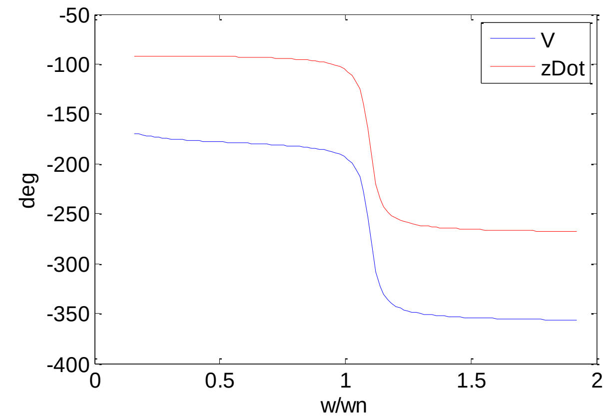
Phase Relationship Between Voltage and Velocity



1/RC >> wn



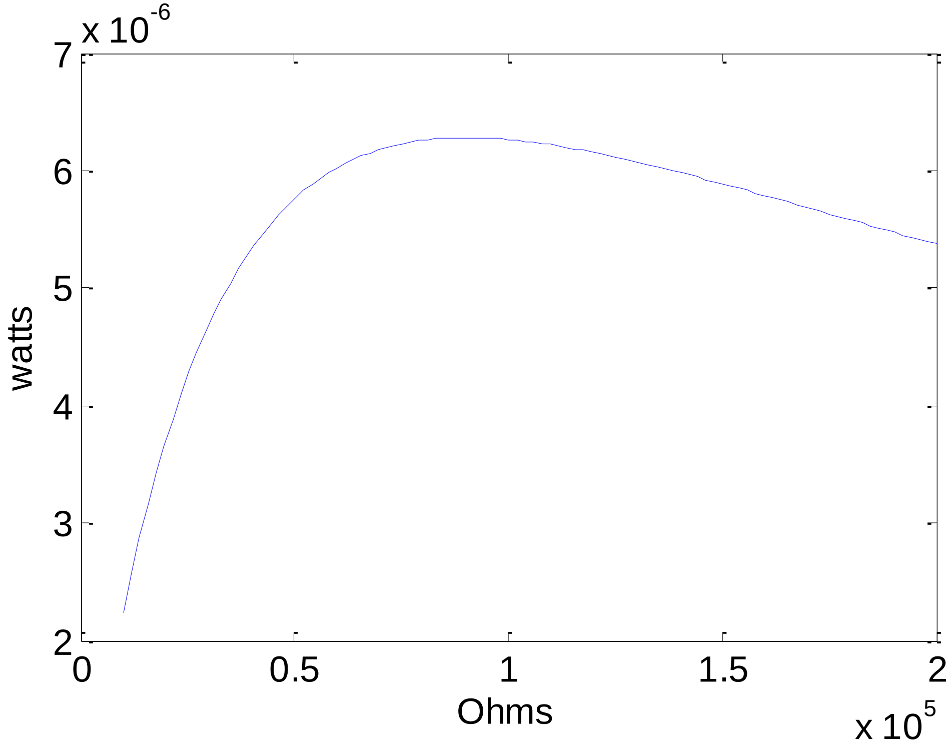
1/RC = wn
V lags by about 45°



1/RC << wn
V lags by about 90°

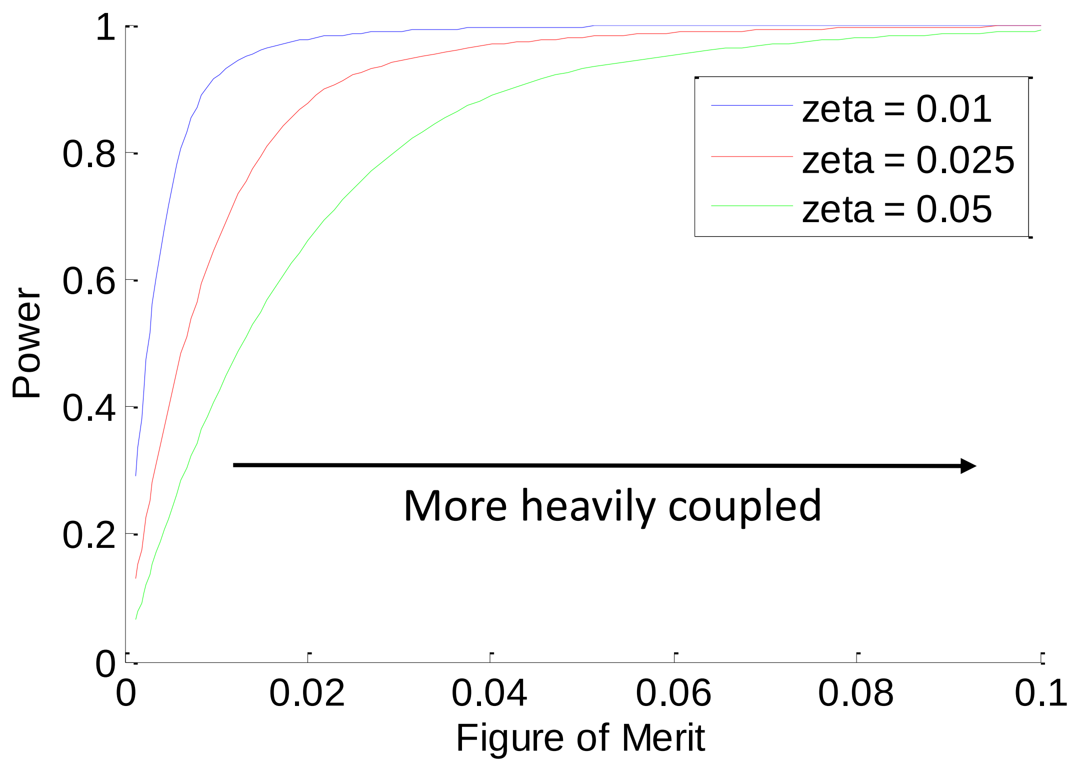
- If piezoelectric coupling could accurately be modeled by a simple viscous damper, voltage and velocity would be exactly in phase as power would be proportional to velocity squared

Power vs.Load Resistance



- Taken at $\omega = \omega_n$
- There is another optimal load resistance at $\omega > \omega_n$
- It’s better to miss on the high side of the effective load resistance than on the low side

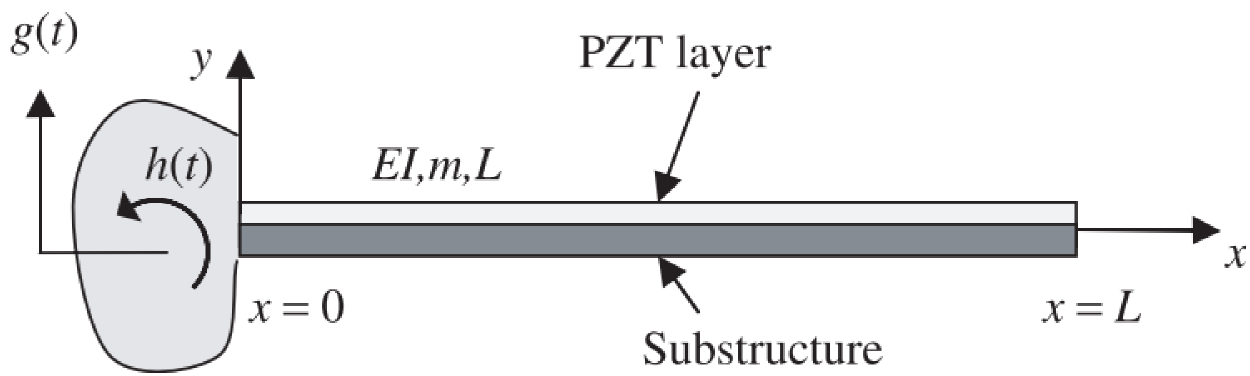
Piezoelectric Figure of Merit and Damping



- Resonance is assumed
- For resonant systems, better piezoelectric materials may not make a large difference
- More coupling results in lower displacement (i.e. more damping)
- Once the proof mass displacement is limited by the electromechanical coupling as opposed to mechanical damping, better material no longer helps much

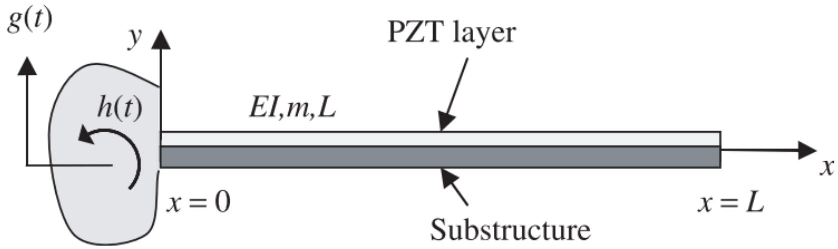
Continuum Modeling

- Problems with lumped parameter model
  - Can lead to inaccuracies especially near resonance if proof mass is on the same order of magnitude as beam mass
  - Only takes first vibration mode into account
- To correct, use Euler-Bernoulli beam model



Erturk and Inman, 2008

Continuum Modeling



Erturk and Inman, 2008

Instead of $m\ddot{z} + b\dot{z} + kz = -mA$ governing the beam mechanics …

Euler Bernoulli beam model

$$EI\,\frac{\partial^4 w_{rel}(x,t)}{\partial x^4} + c_s I\,\frac{\partial^5 w_{rel}(x,t)}{\partial^4 \partial t} + c_a\,\frac{\partial w_{rel}(x,t)}{\partial t} + m\,\frac{\partial w_{rel}(x,t)}{\partial t^2}$$
$$= -m\,\frac{\partial^2 w_b(x,t)}{\partial t^2} - c_a\,\frac{\partial w_b(x,t)}{\partial t}$$

Where w_{rel} = transverse displacement of beam (z-direction) at location x, w_b = displacement of base of beam, EI = flexural stiffness, I = area moment of inertial, c_s = strain rate damping, c_a = viscous damping, m = mass of beam per unit length

*Note the change in notation

Errors Due to Lumped Parameter Model

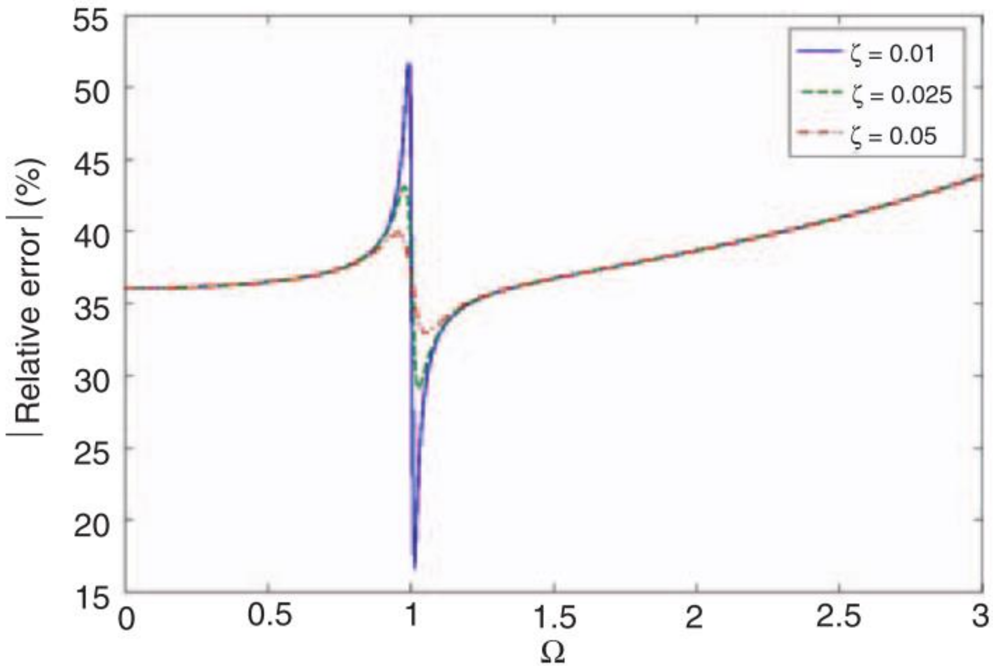


Figure 5. *Error in the relative motion transmissibility due to using the SDOF model for a cantilevered beam without a tip mass in transverse vibrations.*

Erturk and Inman, 2008

- Error for cantilever beam with no proof mass
- For lightly damped beams, large error exists near resonance
- Necessitates the use of a correction factor

Errors Due to Lumped Parameter Model

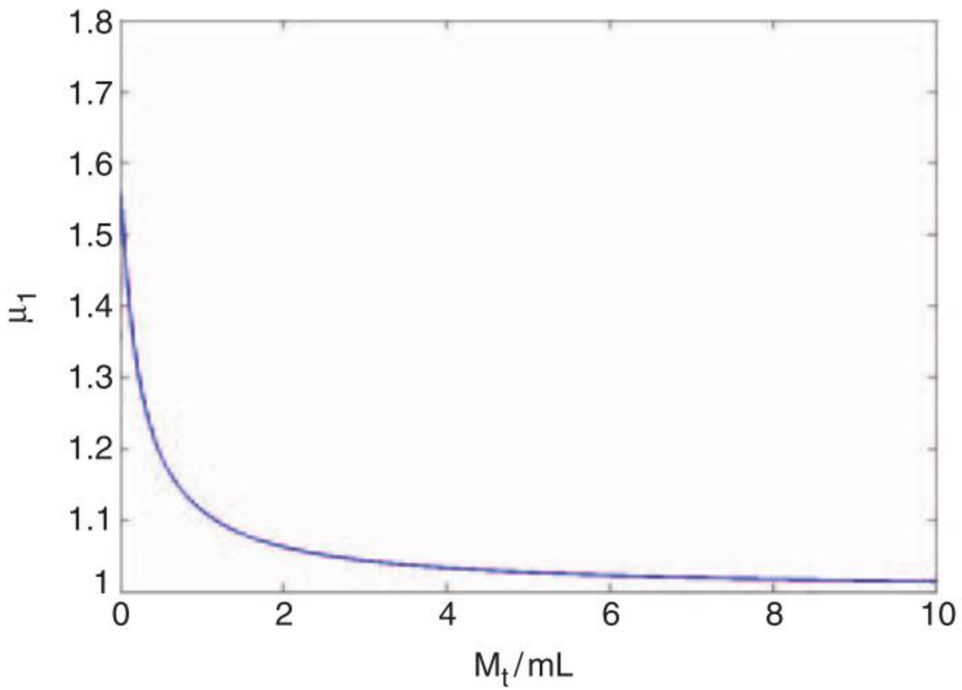


Figure 7. *Variation of the correction factor for the fundamental transverse vibration mode with tip mass to beam mass ratio.*

Erturk and Inman, 2008

- Note that the correction factor is only significant if the proof (or tip) mass is of the same order of magnitude or smaller than the beam mass
- As power scales with mass, most designs incorporate large proof masses, and thus lumped parameter modeling provides reasonably accurate results

Summary

- Piezoelectric harvesters can be more easily miniaturized than electromagnetic harvesters
- At cm^3 size scales, there isn't much difference in performance between electromagnetic and piezoelectric generators
  - Performance is really about getting the right amount of coupling (i.e. electrical damping)
- Voltages tend to be at a good usable level (1-10 volts)
- Source impedance is relatively high (100's of kΩ to a few MΩ)

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